# Beauty of Mathematics Decal PSET #2

Due 09/27

Recall that an infinite quantity is said to be "countable" if it is the same size as the whole numbers $\{0, 1, 2, 3, \ldots\}$ and "uncountable" if it is bigger (i.e. not countable).

1. Show that an uncountable set can't be made up of two countable sets. For example, if we have a countable subset of the real numbers, then the rest of the real numbers must be uncountable, since the whole set of real numbers is uncountable. (Hint: the even numbers and the odd numbers are countable, and they make up the whole numbers, which are also countable).

2. Suppose we have an "alphabet" consisting of finitely many symbols. How many "sentences" are there, if a "sentence" is a finite string of symbols from our alphabet? (Possible hint: alphabetical order).

3. Call a real number *computable* if it's possible to write down a (finite) program which, when run, will output the digits of this number. For a trivial example, the number 1 is computable, because our program can just be `output 1`. For a less trivial example, the number $0.101001000100001\cdots$ is computable, because we could write something like `for each whole number n ≥ 1, output a 1 followed by n zeros`. How many real numbers are computable? How many real numbers are not computable? (Hint: use the previous exercise for the first question and the previous previous exercise for the second question).