

Beauty of Mathematics Decal PSET #1

1. A very special island is inhabited only by knights and knaves. Knights always tell the truth, knaves always lie.

You meet six inhabitants: Betty, Rex, Peggy, Carl, Zeke and Marge.

Betty claims that Marge is a knave.

Rex tells you that Zeke could say that Peggy is a knight.

Peggy tells you, "Only a knave would say that Marge is a knave."

Carl tells you that either Marge is a knight or Peggy is a knight.

Zeke tells you, "Neither Marge nor Rex are knights."

Marge claims that it's not the case that Rex is a knave.

Who are the knights and who are the knaves?

2. Prove the following statement:

"For every integer x , if x is even, then for every integer y , xy is even."

3. (Tougher!) Explain what is wrong with the following argument:

$$0 = 0$$

$$\therefore 0 = 0 + 0 + 0 + 0 + \dots$$

$$\therefore 0 = (1-1) + (1-1) + (1-1) + \dots$$

$$\therefore 0 = 1 - 1 + 1 - 1 + 1 - 1 + \dots$$

$$\therefore 0 = 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \dots$$

$$\therefore 0 = 1 + 0 + 0 + 0 + \dots$$

$$\therefore 0 = 1.$$

(The " \therefore " means "therefore" and " \dots " means continue infinitely many times.)