Worksheet 6 Solutions

MATH 1A Fall 2015

for 27 October 2015

These problems are taken from a set of science problems for calculus written by Jim Belk, available at math.bard.edu/belk/writing.htm. If you're looking for more practice on related rates or exponential growth, check it out! His problems are less terminally boring than the textbook's problems.

Exercise 6.1. In chemistry and physics, *Boyle's Law* describes the relationship between the pressure and volume of a fixed quantity of gas maintained at a constant temperature. The law states that:

$$P\,V \;=\; \text{a constant}$$

where P is the pressure of the gas, and V is the volume.

1. Take the derivative of Boyle's law to find an equation relating $\frac{dP}{dt}$, $\frac{dV}{dt}$, P, and V.

2. A sample of gas is placed in a cylindrical piston. At the beginning of the experiment, the gas occupies a volume of 250 cm^3, and has a pressure of 100 kPa. The piston is slowly compressed, decreasing the volume of the gas at a rate of 50 cm^3/min. How quickly will the pressure of the gas initially increase?

Solution. For the first question, by taking an implicit derivative (using the product rule on the left hand side) we find

$$\frac{dP}{dt}V + P\frac{dV}{dt} = 0.$$

For the second question, we simply plug the values provided into the equation we've just found,

$$\frac{dP}{dt}(250) + (100)(50) = 0,$$

and solve to find $\frac{dP}{dt} = 20$ kPa/min. □

Exercise 6.2. In chemistry, the pH of a solution is defined by the formula

$$\text{pH} \;=\; -0.4343\,\ln(a),$$

where a is the hydrogen ion activity (a measure of the "effective concentration" of hydrogen ions).

1. Suppose the hydrogen ion activity of a solution is increasing at a rate of 0.003/min. How quickly is the pH decreasing when the hydrogen ion activity is 0.02?

2. Suppose instead that the pH of a solution is increasing at a rate of 0.5/min. How quickly is the hydrogen ion activity changing when the pH is 2.5? (Note whether it is increasing or decreasing).

Solution. Taking the implicit derivative of $\text{pH} \;=\; -0.4343\,\ln(a)$, we find

$$\frac{d\text{pH}}{dt} = -\frac{0.4343}{a}\frac{da}{dt}.$$

For the first problem, we can simply plug the values provided into this equation to find

$$\frac{d\text{pH}}{dt} = -\frac{0.4343}{0.02}(0.003) \approx -0.065/\text{min}.$$

For the second problem, we must first use the original equation to find a:

$$2.5 \;=\; -0.4343\,\ln(a),$$

so $a = e^{-2.5/0.4343}$. Now we can plug into the formula we found,

$$0.5 = -\frac{0.4343}{e^{-2.5/0.4343}}\frac{da}{dt},$$

and solve to find

$$\frac{da}{dt} = -\frac{0.5e^{-2.5/0.4343}}{0.4343} \approx -0.003.$$

Note that since the derivative is negative, a is decreasing. □