Worksheet 6

MATH 1A Fall 2015

for 27 October 2015

These problems are taken from a set of science problems for calculus written by Jim Belk, available at math.bard.edu/belk/writing.htm. If you're looking for more practice on related rates or exponential growth, check it out! His problems are less terminally boring than the textbook's problems.

Exercise 6.1. In chemistry and physics, *Boyle's Law* describes the relationship between the pressure and volume of a fixed quantity of gas maintained at a constant temperature. The law states that:

$$PV = \text{a constant}$$

where P is the pressure of the gas, and V is the volume.

1. Take the derivative of Boyle's law to find an equation relating $\frac{dP}{dt}$, $\frac{dV}{dt}$, P, and V.

2. A sample of gas is placed in a cylindrical piston. At the beginning of the experiment, the gas occupies a volume of 250 cm^3, and has a pressure of 100 kPa. The piston is slowly compressed, decreasing the volume of the gas at a rate of 50 cm^3/min. How quickly will the pressure of the gas initially increase?

Exercise 6.2. In chemistry, the pH of a solution is defined by the formula

$$\text{pH} = -0.4343 \ln(a),$$

where a is the hydrogen ion activity (a measure of the "effective concentration" of hydrogen ions).

1. Suppose the hydrogen ion activity of a solution is increasing at a rate of $0.003/\text{min}$. How quickly is the pH decreasing when the hydrogen ion activity is 0.02?

2. Suppose instead that the pH of a solution is increasing at a rate of $0.5/\text{min}$. How quickly is the hydrogen ion activity changing when the pH is 2.5? (Note whether it is increasing or decreasing).