Worksheet 5 Solutions

MATH 1A Fall 2015

for 20 October 2015

Exercise 5.1. Find $\frac{d}{dx} \arcsin x$. [Hint: it may be helpful to use implicit differentiation.]

Solution. We can set $y = \arcsin x$, and taking sin of both sides $\sin y = x$. Now using implicit differentiation,

$$\begin{aligned}
\frac{d}{dx} \sin y &= \frac{d}{dx} x \\
\cos(y) \frac{dy}{dx} &= 1 \\
\frac{dy}{dx} &= \frac{1}{\cos y} \\
\frac{dy}{dx} &= \frac{1}{\cos(\arcsin x)} \\
\frac{dy}{dx} &= \frac{1}{\sqrt{\cos(\arcsin x)^2}} \\
\frac{dy}{dx} &= \frac{1}{\sqrt{1 - \sin(\arcsin x)^2}} \\
\frac{dy}{dx} &= \frac{1}{\sqrt{1 - x^2}}.
\end{aligned}$$

□

Exercise 5.2. This is a problem that I have actually done with some friends in the past.

Berkeley campus is 178 acres, or about 7,750,000 square feet, and let's say a sheep occupies about 10 square feet. Suppose today (or at $t = 0$ or whatever) we buy two sheep, and suppose the reproductive cycle of these sheep lasts a year (gestation plus reaching maturity should only take about 5 + 6 months, but most sheep are seasonal breeders, so we can assume they breed once a year). Most sheep also have litters of 1-2 lambs, so let's say the number of sheep doubles each reproductive cycle (there will be some twins and some single lambs, but also maybe we have less rams than ewes; whatever).

1. Write a model for the number of sheep y we have as a function of time t (and maybe put time in units of years).
2. How long will it take before we can completely cover Berkeley campus with sheep?
3. Suppose I want to cover Berkeley with sheep before I graduate in 5 years. What's the least number of sheep I'd need to start with?

Proof. 1. We want a model of the form $y = ce^{kt}$, or writing $r = e^k$, alternatively $y = cr^t$. Since anything to the power 0 is 1, the constant c must be out initial value, i.e. $c = y(0) = 2$. Also after 1 year we should have 4 sheep (after one year double our initial value), so $4 = 2r^1$, and we find $r = 2$. Thus $y = 2 \cdot 2^t$.

2. We'll need $\frac{7750000}{10} = 775000$ sheep, so we want to solve $775000 = 2 \cdot 2^t$. Dividing by 2 and then taking log base 2, we find

$$t = \log_2(775000/2) = \frac{\ln(775000/2)}{\ln 2} \approx 18.6 \text{ years.}$$

3. If we want to find the number of starting sheep we need in order to get 775000 after 5 years, that means solving $775000 = c2^5$, and we find $c = 775000/2^5 \approx 24219$ sheep. $\square$