Worksheet 4 Solutions

MATH 1A Fall 2015

for 29 September 2015

Exercise 4.1. Prove that there is a real number x for which $\cos x = x$. (You may assume that cos is continuous).

Proof. We're trying to show that a particular equation has a solution, so we should think of the intermediate value theorem.

Consider the function $f(x) = \cos(x) - x$. Note that $f(0) = \cos 0 - 0 = 1$, which is positive, and $f(\pi) = \cos \pi - \pi = -1 - \pi$, which is negative. Note also that f is continuous. By the intermediate value theorem, we conclude that there is a $c \in (0, \pi)$ such that $f(c) = 0$, and thus $\cos c = c$. □

Derivatives! We're interested in understanding functions, and the idea of a derivative is a very powerful tool for this. When a function is differentiable, understanding the derivative gives us lots of information about the function; or maybe we're just interested in understanding the derivative to begin with. As we learn more about derivatives I'll try to give more of an indication why they're awesome.

Exercise 4.2. State the definition of the derivative.

Solution. The derivative of a function $f(x)$ at a is

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

□

Exercise 4.3. For $c \in \mathbb{R}$, prove that the derivative of $f(x) = cx$ is the constant function $f'(x) = c$.

Proof. The derivative of $f(x) = cx$ at a number x is

$$\begin{aligned}
&\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{c(x+h) - cx}{h} \\
&= \lim_{h \to 0} \frac{cx + ch - cx}{h} \\
&= \lim_{h \to 0} \frac{ch}{h} \\
&= \lim_{h \to 0} c \\
&= c.
\end{aligned}$$

□