Worksheet 3

MATH 1A Fall 2015

for 22 September 2015

The squeeze theorem is a useful tool for evaluating limits, and to use it properly it probably helps to understand what it is. It's also useful to be able to translate back and forth between intuitive ways of saying things and precise ways of saying things (and this example in particular will be necessary when we're proving the squeeze theorem or applying it to prove limits).

Exercise 3.1. State the squeeze theorem. Next, if your statement includes (anything like) the phrase "when x is near a, except possibly at a, we have $f(x) \leq g(x) \leq h(x)$", replace this intuitive phrase with a precise mathematical statement.

Exercise 3.2. Prove that

$$\lim_{x \to 0} x^2 e^{\sin x} = 0.$$

(Recall e is just some real number, about 2.7ish).

We'll probably also need to be able to use all sorts of different limits, so here is one to exercise a different definition of limit than the usual one.

Exercise 3.3. For a function $f(x)$, define what it means to say

$$\lim_{x \to a^+} f(x) = \infty.$$

Then prove that

$$\lim_{x \to 0^+} \frac{1}{x} = \infty.$$