Quiz 7 Solutions

MATH 1A Fall 2015

29 October 2015

Exercise 7.1. The hour hand of Big Ben is 9 feet long, and the minute hand is 14 feet long. Find the rate of change of the distance between the tip of the hour hand and the tip of the minute hand at 1pm. [Hint: It may be helpful to think of the hour hand and minute hand as two sides of a triangle, and use the law of cosines: if a triangle has sides a, b, c and the angle between a and b is θ, then $c^2 = a^2 + b^2 - 2ab\cos\theta$.]

Solution. Consider the triangle formed by the two hands of the clock (with the third edge being the line between their tips). Call the length of this third edge c. We want to find $\frac{dc}{dt}$ (at 1pm).

Call the angle between the hands θ. Then by the law of cosines,

$$c^2 = 14^2 + 9^2 - 2(14)(9)\cos\theta.$$

Taking an implicit derivative, we find

$$2c\frac{dc}{dt} = 2(14)(9)\sin(\theta)\frac{d\theta}{dt},$$

or

$$\frac{dc}{dt} = \frac{(14)(9)}{c}\sin(\theta)\frac{d\theta}{dt}.$$

Thus to find $\frac{dc}{dt}$, we need to find c, θ, and $\frac{d\theta}{dt}$.

Since the time is 1pm, the minute hand points to 12 (straight up) and the hour hand points to 1, so the angle between the hands is $1/12^{\text{th}}$ of the circle, or $\theta = \pi/6$ radians. Plugging this into the law of cosines, we find

$$c = \sqrt{14^2 + 9^2 - 2(14)(9)\frac{\sqrt{3}}{2}}.$$

To find $\frac{d\theta}{dt}$, note that the minute hand moves clockwise at 2π radians per hour (because it completes a full circle in an hour) and the hour hand moves clockwise at $\pi/6$ radians per hour (because it moves $1/12$ of a full circle in an hour). Thus $\frac{d\theta}{dt} = \pi/6 - 2\pi$, because the motion of the hour hand increases the angle and the motion of the minute hand decreases the angle.

Now we have all the values we need, and we can plug everything into our equation for $\frac{dc}{dt}$ to find

$$\frac{dc}{dt} = \frac{(14)(9)}{\sqrt{14^2 + 9^2 - 2(14)(9)\frac{\sqrt{3}}{2}}}\frac{1}{2}(\pi/6 - 2\pi),$$

which comes out to about -47 feet per hour. □