Quiz 5 Solutions

MATH 1A Fall 2015

15 October 2015

Exercise 5.1. Consider the equation

$$e^{xy} = 2x \sin y.$$

Find $\frac{dy}{dx}$.

Solution. By implicit differentiation we find

$$\begin{aligned} \frac{d}{dx} e^{xy} &= \frac{d}{dx} 2x \sin y \\ e^{xy} \left(y + x \frac{dy}{dx} \right) &= 2 \sin y + 2x \cos(y) \frac{dy}{dx}, \end{aligned}$$

where we've used the chain rule and product rule on the left-hand side, and the product rule on the right-hand side. Solving for $\frac{dy}{dx}$ gives

$$\begin{aligned} ye^{xy} + xe^{xy} \frac{dy}{dx} &= 2 \sin y + 2x \cos(y) \frac{dy}{dx} \\ xe^{xy} \frac{dy}{dx} - 2x \cos(y) \frac{dy}{dx} &= 2 \sin y - ye^{xy} \\ \frac{dy}{dx} &= \frac{2 \sin y - ye^{xy}}{xe^{xy} - 2x \cos y}. \end{aligned}$$

An alternative form of the solution can be obtained by substituting e^{xy} with $2x \sin y$, because by the original equation these are equal. This results in

$$\frac{dy}{dx} = \frac{2 \sin y - 2xy \sin y}{2x^2 \sin y - 2x \cos y},$$

and dividing top and bottom by $2x \sin y$, we obtain

$$\frac{dy}{dx} = \frac{\frac{1}{x} - y}{x - \cot y}.$$

This is the solution that appears if we apply log to both sides of the original equation before taking the implicit derivative; note that the two solutions are in fact the same. □

Exercise 5.2. Suppose $x = f(y)$. Find $\frac{dy}{dx}$ in terms of x, y, and f and its derivatives.

Solution. Taking an implicit derivative,

$$\frac{d}{dx}x = \frac{d}{dx}f(y)$$

$$1 = f'(y)\frac{dy}{dx},$$

so we find

$$\frac{dy}{dx} = \frac{1}{f'(y)}.$$

□