Quiz 4 Solutions

MATH 1A Fall 2015

1 October 2015

Exercise 3.1. Find the derivative of $\tan(1 + x^2)$.

Solution. We have a composition of two familiar functions, namely $\tan x$ and $1 + x^2$, so we'll use the chain rule.

First of all, the derivative of $\tan x$ is $\sec^2 x$, which we can either remember or find using the quotient rule:

$$\begin{aligned} \frac{d}{dx} \tan x &= \frac{d}{dx} \frac{\sin x}{\cos x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

(where we've used the trig identity $\sin^2 x + \cos^2 x = 1$). Now using the chain rule,

$$\frac{d}{dx} \tan(1 + x^2) = \sec^2(1 + x^2) \cdot 2x.$$

□

Exercise 3.2. Prove that the polynomial $x^4 - x - 4$ has a root in the interval $[-2, 2]$.

Proof. Note that $f(x) = x^4 - x - 4$ is continuous. Note also that

$$f(-2) = 14 > 0 \qquad \text{and} \qquad f(0) = -4 < 0.$$

By the intermediate value theorem, there is a $c \in (-2, 0)$ such that $f(c) = 0$. (In particular, this c is also in $[-2, 2]$). □