Quiz 3 Solutions

MATH 1A Fall 2015

24 September 2015

Exercise 3.1. Prove that
$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) + x = 0.$$

Proof. Note that
$$-1 \le \cos\left(\frac{1}{x}\right) \le 1,$$
$$-x^2 \le x^2 \cos\left(\frac{1}{x}\right) \le x^2,$$
$$-x^2 + x \le x^2 \cos\left(\frac{1}{x}\right) + x \le x^2 + x$$
for all $x \in \mathbb{R}$. Note also that
$$\lim_{x\to 0} -x^2 + x = \lim_{x\to 0} x^2 + x = 0.$$
By the squeeze theorem, we conclude
$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) + x = 0.$$
$\square$

Exercise 3.2. State the definitions of the following limits.

$$\lim_{x\to a^-} f(x) = \infty \qquad\qquad \lim_{x\to -\infty} f(x) = L$$

Solution. We say $\lim_{x\to a^-} f(x) = \infty$ if for every $M \in \mathbb{R}$ there is a $\delta > 0$ such that if $a - \delta < x < a$ then $f(x) > M$.

We say $\lim_{x\to -\infty} f(x) = L$ if for every $\varepsilon > 0$ there is an $N \in \mathbb{R}$ such that if $x < N$ then $|f(x) - L| < \varepsilon$. $\square$