

Quiz 2 Solutions

MATH 1A Fall 2015

17 September 2015

Exercise 2.1. State the definition of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ being continuous at a point $a \in \mathbb{R}$.

Proof. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous at $a \in \mathbb{R}$ if

$$\lim_{x \rightarrow a} f(x) = f(a).$$

□

Exercise 2.2. Prove that

$$\lim_{x \rightarrow 1} \frac{x^5 + 3x^2 + 2}{x + 1} = 3.$$

You may assume that polynomials are continuous.

Proof. If you are content to assume that our rational function is continuous, then you can just plug in $x = 1$ to find the limit. If you want to only assume that *polynomials* are continuous, here is a proof:

Since polynomials are continuous, we have

$$\lim_{x \rightarrow 1} x^5 + 3x^2 + 2 = 6 \quad \text{and} \quad \lim_{x \rightarrow 1} x + 1 = 2.$$

Since the denominator is not zero, we can use the limit law for quotients to see

$$\lim_{x \rightarrow 1} \frac{x^5 + 3x^2 + 2}{x + 1} = \frac{6}{2} = 3.$$

□