Practice Final Exam

MATH 1A Fall 2015

Problem 1. A 13 foot ladder rests against a wall. The base of the ladder is pushed toward the wall at 2 feet per second. How fast is the top of the ladder moving up the wall when the base is 5 feet from the wall?

Problem 2. Prove that there is a real number x for which $\ln x = \frac{1}{x}$.

Problem 3. Find the derivatives of the following functions.

(a) $x^2 e^x$

(b) $\ln(\sec x + \tan x)$

(c) x^x

Problem 4. (a) Define what it means to say $\lim_{x \to a^+} f(x) = \infty$.

(b) Prove, using the definition from the previous part, that $\lim_{x \to 2^+} \frac{1}{x-2} = \infty$.

Problem 5. State the extreme value theorem.

Problem 6. (a) State the limit definition of the derivative.

(b) Prove, using the definition from the previous part, that $\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$.

Problem 7. Let f be a differentiable function. Suppose that $f(0) = 0$ and $f'(x) > 0$ for all x. Prove that $f(x) > 0$ for all $x > 0$.

Problem 8. State and prove the squeeze theorem.

Problem 9. Find $\frac{dy}{dx}$ if $y^2 x + \ln y = \sin(2x)$.

Problem 10. Moore's law is the observation that the number of transistors in computer processors has doubled every two years. Suppose a 2011 proccessor has 2.6 billion transistors.

(a) Write a model for the number of transistors in a processor as a function of time.

(b) How many transistors did 1971 processors have?

Problem 11. If two numbers add up to 6, what is the largest their product can be?

Problem 12. State the fundamental theorem of calculus.

Problem 13. Find the antiderivatives of the following functions.

(a) $(x+2)(x+4)$

(b) $\tan x$

(c) $x3^{x^2+3}$

Problem 14. Evaluate the following limits. Show work, but there is no need to justify each step.

(a) $\lim_{x\to\infty} \frac{(x-1)(2x+2)}{x^2+4x+3}$

(b) $\lim_{x\to 0} x^2 \sin(\frac{1}{x})$

(c) $\lim_{x\to 0} \frac{\sin 2x}{x}$

Problem 15. (a) Define what it means to say a function $f(x)$ is continuous at a point a.

(b) Prove, using the definition above, that $f(x) = 3x + 2$ is continuous at 1.

Problem 16. Find the 50^{th} derivative of $f(x) = e^{2x+1}$.

Problem 17. Let P and Q be logical statements, and suppose P **is true** and Q **is false**. Decide whether or not the following statements are true or false.

(a) P and not Q

(b) Q implies P

(c) (not P) if and only if Q