MATH 16B - WORKSHEET 8

1

i Find a polynomial p of degree 3 such that $p(0) = 0$, $p'(0) = 3$, $p''(0) = 0$ and $p'''(0) = -4$. Is it unique?

ii Find a polynomial p of degree 3 such that $p(1) = 0$, $p'(1) = 3$, $p''(1) = 0$ and $p'''(1) = -4$

2

i Find the fourth order Taylor polynomial of $f(x) = xe^x$ at $x = 0$.

ii Find the first four non-zero coefficients of the Taylor expansion of $g(x) = (\sin x)^2$ at $x = 0$.

iii Use the second order and fourth order Taylor polynomials of f and g, to estimate $f(0.5)$ and $g(0.8)$. Compute $g(0.5)$ and $f(0.5)$ using a calculator. Which of the estimates is more precise?.

iv By computing the first few terms of the Taylor expansion of e^{-x^2} at $x = 0$, approximate $\int_0^{\frac{1}{2}} e^{-x^2} dx$. One can show that the actual value is very close to 0.461281; compare this with your approximation.

v Determine the n-th Taylor polynomial of $f(x) = \sqrt{x}$ at $x = 1$.

3 Determine whether the following "formal" sums converge or diverge. If they converge, compute their value.

i $\dfrac{3}{5^3} - \dfrac{3^4}{5^5} + \dfrac{3^7}{5^7} - \dfrac{3^{10}}{5^9} + \dfrac{3^{13}}{5^{11}} - \cdots$

ii $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{1}{5}\right)^{2k}$

iii $1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} + \cdots$

ii $1 - \dfrac{2}{1!} + \dfrac{2^2}{2!} - \dfrac{2^3}{3!} + \dfrac{2^5}{4!} - \cdots$