

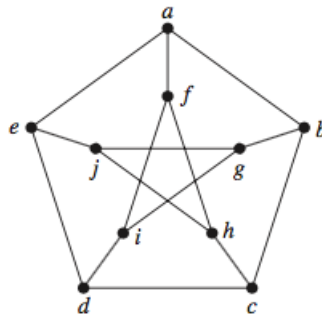
# MATH 55 - WORKSHEET 7 (TUESDAY)

1 For what values of  $m$  and  $n$  does the complete graph  $K_{m,n}$  have an

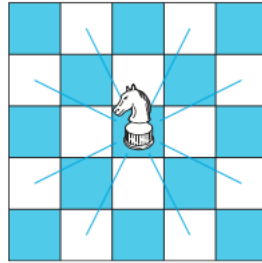
a Euler circuit

b Euler path

2 Show that the Petersen graph, shown below, does not have a Hamilton circuit, but that the subgraph obtained by deleting a vertex  $v$ , and all edges incident with  $v$ , does have a Hamilton circuit.



- 3 A knight is a chess piece that can move either two spaces horizontally and one space vertically or one space horizontally and two spaces vertically. That is, a knight on square  $(x, y)$  can move to any of the eight squares  $(x \pm 2, y \pm 1)$ ,  $(x \pm 1, y \pm 2)$ , if these squares are on the chessboard, as illustrated here:



A **knight's tour** is a sequence of legal moves by a knight starting at some square and visiting each square exactly once.

- a Draw the graph that represents the legal moves of a knight on a  $3 \times 3$  chessboard (account for all possible starting locations).
- b Draw the graph that represents the legal moves of a knight on a  $3 \times 4$  chessboard.
- c Show that finding a knight's tour on an  $m \times n$  chessboard is equivalent to finding a Hamilton path on the graph representing the legal moves of a knight on that board.

**d** Show that there is a knight's tour on a  $3 \times 4$  chessboard.

**e** Show that there is no knight's tour on a  $3 \times 3$  chessboard.

**f** Show that there is a knight's tour on an  $8 \times 8$  chessboard.