MATH 16B - WORKSHEET 7

1

i Find all solutions y to the differential equation $y' = \frac{4}{t}y + t^6$ and $t > 0$.

ii [10.4.25 in the textbook] In one intravenous infusion, morphine was injected continuously at an increasing rate of t milligrams per hour. Suppose that the body removes the drug at a rate proportional to the amount of drug present in the body, with constant of proportionality $k = 0.35$. Let $f(t)$ denote the amount of morphine in the body, t hours from the beginning of the infusion.

a Find a differential equation satisfied by $f(t)$.

b Assuming that the infusion lasted 8 hours, determine the amount of morphine in the body during the infusion if at the beginning of the infusion the body was free of morphine.

ii [10.4.20 in the textbook] Radium 226 is a radioactive substance with a decay constant $.00043$. Suppose that radium 226 is being continuously added to an initially empty container at a constant rate of 3 milligrams per year. Let $P(t)$ denote the number of grams of radium 226 remaining in the container after t years.

a Find an initial-value problem satisfied by $P(t)$.

b Solve the initial-value problem for $P(t)$.

c What is the limit of the amount of radium 226 in the container as t tends to infinity.

2

i Compute $\int \tan x dx$.

ii Find the solution to the initial value problem, $y' \cos x = -y \sin x + 4 \sin x \cos x - (\cos x)^2$, $y(0) = 1$ for $0 \leq x \leq \frac{\pi}{2}$.

3

i [10.5.11 in the textbook] Sketch the graph of the solutions to $y' = -\frac{1}{3}(y+2)(y-4)$ that satisfy $y(0) = -3$, $y(0) = -1$ and $y(0) = 6$.

ii [10.5.31 in the textbook] Sketch the graph of the solutions to $y' = \frac{1}{y}$ that satisfy $y(0) = -1$ and $y(0) = 1$.