

MATH 55 - WORKSHEET 6 (THURSDAY - FRIDAY)

- 0 Read the “New Graphs from Old” section in the book (pages 663 – 664); I will do a brisk review of this, next lecture.
- 1 Let G be a graph in which every vertex has degree 2. Prove that G is a union of cycles.
- 2 What are the sizes of the maximum matchings in $K_{m,n}$, K_n , C_n , W_n ? Your answers will depend on n, m .
- 3 Find a counterexample to Hall’s marriage theorem when the graph G is *infinite*! In particular, find a bipartite graph $G = (V, E)$ with infinitely many vertices that satisfies $|N(A)| \geq |A|$ for all $A \subseteq V$, but doesn’t have a complete matching.

- 4 A matching M in a graph G is called **perfect** if every vertex in G is incident to an edge in M . Alternatively, a perfect matching is a matching that matches all the vertices in a graph.
- a Show that a graph with a perfect matching has an even number of vertices. For what values of n does C_n have a perfect matching?
- b Two people are playing the following game on a graph G : One after another the players choose vertices (one per turn) v_1, v_2, v_3, \dots so that v_{i+1} is adjacent to v_i for all $i \geq 1$. The last player which is able to choose a vertex wins. Prove that the first player has a winning strategy if and only if the graph has no perfect matching.