MATH 16B - WORKSHEET 6

Note The red font in the question statement denotes portions that differ from the original worksheet; most of these correspond to fixed typos.

1

i Consider the initial-value problem $y' = e^t + 2y$, and $y(0) = 0$. If $f(t)$ was a solution this problem, find $f(0)$ and $f'(0)$.

ii Find all constant solutions of the differential equation $y' = y^3 - 6y^2 + 11y - 6$?

iii Compute all possible values of $f'(0)$ where $f(t)$ is a solution to $\sin(y') + \ln y = t^3 - 4t$ and $y(0) = e^1$.

2 [10.1.17 in the textbook] A certain piece of news is being broadcast to an audience. Let $f(t)$ denote the number of people who have heard the news after t hours. Suppose that $y = f(t)$ satisfies the differential equation $y' = .07(200,000 - y)$, and $y(0) = 10$. Describe the initial-value problem in words.

3 Solve the following differential equations

i $\dfrac{dy}{dt} = -\dfrac{5}{t^2y^2}$

ii $yy' = t\sin(t^2 + 1)$

iii $y' = \dfrac{x^3}{y(1+x^4)}$ with $y(0) = 4$.

iv $y' = \dfrac{t+1}{ty}$ with $t > 0$ and $y(1) = -3$.

4 Assume that Berkeley's population follows the growth model $P' = (at + b)P$ for some constants a, b. The population in 2013 was $116,768$, the population in 1990 was $103,137$ and population in year 0 was 1000. Using these two facts, solve the differential equation to obtain Berkeley's population model, P.