MATH 55 - Worksheet 5 (Thursday)

1 A vending machine dispensing books of stamps accepts only one-dollar coins, \$1 bills, and \$5 bills.

a Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.

b What are the initial conditions?

c How many ways are there to deposit \$10 for a book of stamps?

2a Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive 0s or two consecutive 1s.

b What are the initial conditions?

c How many ternary strings of length six do not contain two consecutive 0s or two consecutive 1s?

3a Find a recurrence relation for the number of ways to completely cover a $2 \times n$ checkerboard with 1×2 dominoes. [Hint: Consider separately the coverings where the position in the top right corner of the checkerboard is covered by a domino positioned horizontally and where it is covered by a domino positioned vertically.]

b What are the initial conditions for the recurrence relation in part a?

c How many ways are there to completely cover a 2×17 checkerboard with 1×2 dominoes

4 A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

a Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n, under the assumption for this model.

b Find L_n if $100,000$ lobsters were caught in year 1 and $300,000$ were caught in year 2.

2 Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$.

5 Consider the non-homogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$

a Show that $a_n = -2^{n+1}$ is a solution to this recurrence relation.

b Find all solutions of this recurrence relation.

c Find the solution with $a_0 = 1$.