MATH 55 - WORKSHEET 5 (TUESDAY)

1 Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains two white balls and three blue balls, and the second box contains four white balls and one blue ball. What is the probability that Frida picked a ball from the first box if she has selected a blue ball?

2 When a test for steroids is given to soccer players, 98% of the players taking steroids test positive and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids?

3 In this exercise we will use Bayes' theorem to solve the Monty Hall puzzle. In this puzzle you are asked to select one of three doors to open. There is a large prize behind one of the three doors and the other two doors are losers. After you select a door, Monty Hall opens one of the two doors you did not select that he knows is a losing door, selecting at random if both are losing doors. Monty asks you whether you would like to switch doors. Suppose that the three doors in the puzzle are labeled 1, 2, and 3. Let W be the random variable whose value is the number of the winning door; assume that $p(W = k) = \frac{1}{3}$ for $k = 1, 2, 3$. Let M denote the random variable whose value is the number of the door that Monty opens. Suppose you choose door i.

a What is the probability that you will win the prize if the game ends without Monty asking you whether you want to change doors?

b Find $p(M = j | W = k)$ for $j = 1, 2, 3$ and $k = 1, 2, 3$.

c Use Bayes' theorem to find $p(W = j|M = k)$ where i, j and k are distinct values.

d Explain why the answer to tells you whether you should change doors when Monty gives you the chance to do so.

4 Ramesh can get to work in three different ways: by bicycle, by car, or by bus. Because of commuter traffic, there is a 50% chance that he will be late when he drives his car. When he takes the bus, which uses a special lane reserved for buses, there is a 20% chance that he will be late. The probability that he is late when he rides his bicycle is only 5%. Ramesh arrives late one day. His boss wants to estimate the probability that he drove his car to work that day. Suppose the boss knows that Ramesh drives 30% of the time, takes the bus only 10% of the time, and takes his bicycle 60% of the time. What estimate for the probability that Ramesh drove his car does the boss obtain from Bayes' theorem using this information?

5 Prove the extended form of Bayes' theorem. That is, suppose that E is an event from a sample space S and that F_1, F_2, ..., F_n are mutually exclusive events such that $\bigcup_{i=1}^{n} F_i = S$. Assume that $p(E) \neq 0$ and $p(F_i) \neq 0$ for $i = 1, 2, \ldots, n$. Show that

$$p(F_j|E) = \frac{p(E|F_j)p(F_j)}{\sum_{i=1}^{n} p(E|F_i)p(F_i)}$$

[Hint: Use the fact that $E = \bigcup_{i=1}^{n}(E \cap F_i)$.]