

MATH 55 - WORKSHEET 5 (MONDAY)

- 1 A deck of Poker cards contains 52 cards. There are 13 different kinds of cards, with four cards of each kind. These kinds are twos, threes, fours, fives, sixes, sevens, eights, nines, tens, jacks, queens, kings, and aces. There are also four suits: spades, clubs, hearts, and diamonds, each containing 13 cards, with one card of each kind in a suit. What is the probability that a card selected at random from a standard deck of 52 cards is an ace or a heart?
- 2 What is the probability that a five-card poker hand contains a straight flush, that is, five cards of the same suit of consecutive kinds?
- 3 Find the probability of selecting none of the correct six integers in a lottery, where the order in which these integers are selected does not matter, from the positive integers not exceeding 40.
- 4 What is the probability that a player of a lottery wins the prize offered for correctly choosing five (but not six) numbers out of six integers chosen at random from the integers between 1 and 40, inclusively?
- 5 Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?
- 6 Find the probability of each outcome when a loaded die is rolled, if a 3 is twice as likely to appear as each of the other five numbers on the die.

- 7 Show that if E_1, E_2, \dots, E_n are events from a finite sample space, then $p(E_1 \cup \dots \cup E_n) \leq p(E_1) + \dots + p(E_n)$. This is known as Boole's inequality. (Hint: Use induction on n).
- 8 Show that if E and F are independent events, then \bar{E} and \bar{F} are also independent events.
- 9 Find the smallest number of people you need to choose at random so that the probability that at least one of them has a birthday today exceeds $\frac{1}{2}$.
- 10 Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent?
- 11 Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if
- a a 0 bit and a 1 bit are equally likely.
 - b the probability that a bit is a 1 is 0.6.
 - c the probability that the i -th bit is a 1 is $\frac{1}{2^i}$ for $i = 1, 2, 3, \dots, 10$