

## MATH 55 - WORKSHEET 5 (FRIDAY)

1 Find a closed form for the generating function for the sequence  $\{a_n\}$ , where

a  $a_n = 3^n$  for all  $n \geq 0$

b  $a_n = \binom{8}{n}$  for all  $n \geq 0$

2 Find the coefficient of  $x^{12}$  in the power series of each of these functions

a  $1/(1 + 3x)$

b  $1/(1 - 2x)^2$

c  $x^3/(1 + 4x)^2$

- 3 Use generating functions to find the number of ways to choose a dozen bagels from three varieties - egg, salty, and plain - if at least two bagels of each kind but no more than three salty bagels are chosen.
- 4 Use generating functions to solve the recurrence relation  $a_k = 7a_{k-1}$  with the initial condition  $a_0 = 5$ .
- 5 Use generating functions to prove Vandermonde's identity:  $C(m+n, r) = \sum_{k=0}^r C(m, r-k)C(n, k)$ , whenever  $m, n$  and  $r$  are nonnegative integers with  $r$  not exceeding either  $m$  or  $n$ . [Hint: Look at the coefficient of  $x^r$  in both sides of  $(1+x)^{m+n}$  and  $(1+x)^m(1+x)^n$ ]