

MATH 55 - WORKSHEET 4 (WEDNESDAY)

- 1 A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
 - a How many balls must she select to be sure of having at least three balls of the same color?

 - b How many balls must she select to be sure of having at least three blue balls?

- 2 How many numbers must be selected from the set $1, 3, 5, 7, 9, 11, 13, 15$ to guarantee that at least one pair of these numbers add up to 16?

- 3 The Ramsey number $R(m, n)$, where m and n are positive integers greater than or equal to 2, denotes the minimum number of people at a party such that there are either m mutual friends or n mutual enemies, assuming that every pair of people at the party are friends or enemies.
 - a Show that in a group of five people (where any two people are either friends or enemies), there are **not** necessarily three mutual friends or three mutual enemies. Using this, together with the example in lecture ([Example 13](#) on page 404 in the book), conclude that $R(3, 3) = 6$.

 - b Show that if n is an integer with $n \geq 2$, then the Ramsey number $R(2, n)$ equals n .

- 4 There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusively. Show that at least two houses have addresses that are consecutive integers.
- 5 [Challenge] Suppose that 21 girls and 21 boys enter a mathematics competition. Furthermore, suppose that each entrant solves at most six questions, and for every boy-girl pair, there is at least one question that they both solved. Show that there is a question that was solved by at least three girls and at least three boys.