MATH 55 - WORKSHEET 4 (THURSDAY)

1 How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?

2 How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

3 The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

a exactly one vowel?

b exactly two vowels?

c at least one vowel?

d at least two vowels?

4 How many bit strings contain exactly eight 0s and ten 1s if every 0 must be immediately followed by a 1?

5 A **circular r-permutation of** n people is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.

a Find the number of circular 3-permutations of 5 people.

b Find a formula for the number of circular r-permutations of n people.

6 How many ways are there for a horse race with four horses to finish if ties are possible? [Note: Any number of the four horses may tie.]

7 Find the coefficient of x^5y^8 in $(x+y)^{13}$

8 Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$

a Using a combinatorial argument

b By algebraic manipulation