MATH 55 - WORKSHEET 4 (MONDAY)

1 Find $f(2)$, $f(3)$, $f(4)$, and $f(5)$ if f is defined recursively by $f(0) = -1$, $f(1) = 2$, and for $n = 1, 2, \ldots$

a $f(n+1) = f(n)$

b $f(n+1) = f(n)^2 f(n-1)$

c $f(n+1) = 3f(n)^2 - 4f(n-1)^2$

d $f(n+1) = \frac{f(n-1)}{f(n)}$

2 Give a recursive definition of the sequence $\{a_n\}$ for $n = 1, 2, 3, \ldots$ if

a $a_n = 4n - 2$

b $a_n = 1 + (-1)^n$

c $a_n = n(n+1)$

d $a_n = n^2$

3 Let f_n be the n-th Fibonacci number. Prove that $f_1 + f_3 + \cdots + f_{2n-1} = f_{2n}$ for any $n \in \mathbf{Z}^+$.

4 Give a recursive definition of each of these sets of ordered pairs of positive integers. [Hint: Plot the points in the set in the plane and look for lines containing points in the set]

a $S = \{(a, b) : a \in \mathbf{Z}^+, b \in \mathbf{Z}^+, \text{ and } a + b \text{ is odd}\}$

b $S = \{(a, b) : a \in \mathbf{Z}^+, b \in \mathbf{Z}^+, 3|(a + b)\}$

5 Use structural induction to show that $n(T) \geq 2h(T) + 1$, where T is a full binary tree, $n(T)$ equals the number of vertices of T, and $h(T)$ is the height of T.