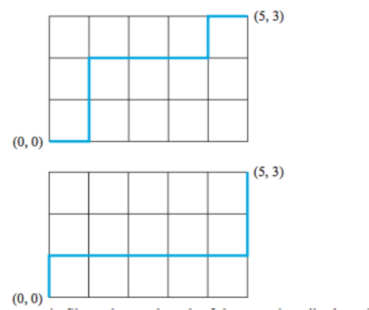
MATH 55 - WORKSHEET 4 (FRIDAY)

1 Prove the binomial theorem using mathematical induction.

2 In this exercise we will count the number of paths in the xy plane between the origin $(0,0)$ and point (m,n), where m and n are nonnegative integers, such that each path is made up of a series of steps, where each step is a move one unit to the right or a move one unit upward. (No moves to the left or downward are allowed.) Two such paths from $(0,0)$ to $(5,3)$ are illustrated here.



a Show that each path of the type described can be represented by a bit string consisting of m 0s and n 1s, where a 0 represents a move one unit to the right and a 1 represents a move one unit upward.

b Conclude from part a that there are $\binom{m+n}{n}$ paths of the desired type.

3 Use the previous problem to give an alternative proof of Corollary 2 in Section 6.3, which states that $\binom{n}{k} = \binom{n}{n-k}$ whenever k is an integer with $0 \leq k \leq n$ [Hint: Consider the number of paths from $(0,0)$ to $(n-k,k)$ and from $(0,0)$ to $(k,n-k)$]

4 Use Exercise 2 to prove Pascal's identity. [Hint: Show that a path from $(0,0)$ to $(n+1-k,k)$ passes through either $(n+1-k,k-1)$or $(n-k,k)$, but not through both.]

5 How many ways are there to assign three jobs to five employees if each employee can be given more than one job?

6 How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels?

7 How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$ where the x_i are non-negative integers such that

a $x_i > 1$ for all $i \in \{1, \ldots, 6\}$?

b $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$, $x_5 \geq 5$ and $x_6 \geq 6$?

c $x_1 \leq 5$?

a $x_1 < 8$ and $x_2 > 8$?

8 How many strings with five or more characters can be formed from the letters in *SEERESS*?

9 In how many ways can a dozen books be placed on four distinguishable shelves

a if the books are indistinguishable copies of the same title?

b if no two books are the same, and the positions of the books on the shelves matter? [Hint: Break this into 12 tasks, placing each book separately. Start with the sequence 1, 2, 3, 4 to represent the shelves. Represent the books by b_i for $i \in \{1, \ldots, 12\}$. Place b_1 to the right of one of the terms in 1, 2, 3, 4. Then successively place b_2, b_3 and b_{12}.]

10 How many ways are there to distributive five indistinguishable objects into three indistinguishable boxes?