MATH 55 - Worksheet 3 (Wednesday)

1 Prove that for any non-negative integer n,

$$2 - 2 \cdot 7 + 2 \cdot 7^2 - 2 \cdot 7^3 + \cdots + 2 \cdot (-7)^n = \frac{1 - (-7)^{n+1}}{4}.$$

2

a Find a formula for $\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n}$ by examining the values of this expression for small values of n.

b Prove the formula you conjectured in part a.

3 Prove that for every integer n,

$$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2.$$

4

a Prove that a set with n elements has $\frac{n(n-1)}{2}$ subsets containing exactly two elements whenever n is an integer greater or equal to 2.

b Prove that a set with n elements has $\frac{n(n-1)(n-2)}{6}$ subsets containing exactly three elements whenever $n \geq 3$.

5 What's wrong with this "proof"?

"Theorem": For every positive integer n, if x and y are positive integers with $\max(x, y) = n$, then $x = y$.

Basis Step: Suppose that $n = 1$. If $\max(x, y) = 1$ and x, y are positive integers, we have $x = 1$ and $y = 1$.

Inductive Step: Let k be a positive integer and assume that whenever $\max(x, y) = k$ and x, y are integers, then $x = y$. Now let $\max(x, y) = k + 1$, where x and y are positive integers. Then, $\max(x - 1, y - 1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. Thus, $x = y$, completing the inductive step.

6 Use mathematical induction to prove Lemma 3 of Section 4.3 which states that if p is a prime and $p | a_1 a_2 \cdots a_n$, where a_i is an integer for $i = 1, 2, 3 \ldots, n$, then $p | a_i$ for some integer i (You might find Lemma 2 in Section 4.3 useful, which states if a, b, c are positive integers such that $\gcd(a, b) = 1$ and $a | bc$, then $a | c$.