

MATH 55 - WORKSHEET 3 (THURSDAY)

- 1 Use strong induction to show that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps for all $n \geq 18$.
- 2 Use strong induction to show that $\sqrt{2}$ is irrational [Hint: Let $P(n)$ be the statement that $\sqrt{2} \neq \frac{n}{b}$ for any positive integer b]
- 3 A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A move is made each time a piece is added to a block, or when two blocks are joined. Use strong induction to prove that no matter how the moves are carried out, exactly $n - 1$ moves are required to assemble a puzzle with n pieces.

4 **[Pick's Theorem]** The area of a simple polygon P in the plane with vertices that are all lattice points (that is, points with integer coordinates) equals $I(P) + \frac{B(P)}{2} - 1$, where $I(P)$ and $B(P)$ are the number of lattice points in the interior of P and on the boundary of P , respectively.

Prove this theorem using strong induction [Hint: For the basis step, first prove the theorem for rectangles, then for right triangles and finally for all triangles by noting that the area of a triangle is the area of a larger rectangle containing it with the areas of at most three triangles subtracted. For the inductive step, take advantage of [Lemma 1](#) from [Section 5.2](#)]