MATH 55 - WORKSHEET 2 (WEDNESDAY)

1 Convert each of the following numbers into it's binary expansion.

a 321

b 1023

c $(163E2)_{16}$

2 Show that a positive integer is divisible by 3 if and only if the sum of its decimal digits is divisible by 3.

3 Prove that for every positive integer n, there are n consecutive composite integers. [Hint: Consider the n consecutive integers starting with $(n+1)! + 2$.]

Definition Given $a, b \in \mathbf{Z}^+$, the division algorithm states that there exists integers q, r with $0 \leq r < b$ such that $a = bq + r$. We define the quantity $a \bmod b$ to be r.

Example $17 \bmod 3 = 2$ and $9 \bmod 3 = 0$

The importance of this definition is established by the following theorem:

4 Let $m \in \mathbf{Z}^+$, then $a \equiv b \bmod m$ if and only if $a \bmod m = b \bmod m$. Prove this theorem.

By virtue of the above theorem, we can write $2 \equiv 17 \bmod 3$ as $17 \bmod 3 = 2 \bmod 3$. In many arguments, it's convenient to view $a \bmod m$ as an actual integer, rather than something modulo m.

5 Show that if a, b are both positive integers, then $2^a - 1 \bmod (2^b - 1) = 2^{a \bmod b} - 1$. [Hint: $2^b \equiv 1 \bmod (2^b - 1)$]