MATH 55 - WORKSHEET 2 (TUESDAY)

1 Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

a The integers greater than 10

b The odd negative integers

c The integers with absolute value less than $1,000,000$

d The real numbers between 0 and 2

e The set $A \times \mathbf{Z}^+$ where $A = \{1, 2\}$

f The integers that are multiples of 10

2 I proved in class that $|\mathbf{Q}^{>0}| = |\mathbf{Z}^+|$. Use the idea of the proof that $|\mathbf{Z}| = |\mathbf{Z}^+|$, to show that $|\mathbf{Q}| = |\mathbf{Z}^+|$ i.e. $\mathbf{Q}$ is countable.

3 Prove that if a is an integer other than 0,

a 1 divides a.

b a divides 0.

4 Evaluate these quantities.

a $-17 \bmod 2$

b $144 \bmod 7$

c $-101 \bmod 13$

d $199 \bmod 19$

4 Decide whether each of these integers is congruent to 3 modulo 7

a 37

b 66

c -17

d -67

5 Given any integer n show that $n^2 + n \equiv 0 \bmod 2$.

6 Prove that if n is an odd integer, then $n^2 \equiv 1 \bmod 8$. [Hint: Write $n = 2k + 1$ and use Exercise 5]