MATH 16B - WORKSHEET 2

1 Consider $f(x,y) = x^2 + xy - 3y^2$

i Find all relative maxima/minima of $f(x,y)$.

ii Without looking in your notes/textbook describe the method of Lagrange multipliers. Use this to maximize $f(x,y)$ subject to $x + 2y = 2$.

2 Find the values of x, y, z that minimize $x^2 + y^2 + z^2 - 3x - 5y - z$ subject to the constraint $20 - 2x - y - z = 0$.

3 Last week we considered the production function $f(x,y) = 60x^{\frac{3}{4}}y^{\frac{1}{4}}$; recall that x represents the units of labour and y the units of capital. Now assume that each unit of labour costs $\$50$ and each unit of capital costs $\$100$. Also assume that $\$20,000$ is available to spend on production.

i How many units of labor and how many units of capital should be utilized to maximize production? [Hint: The labour cost, capital cost and total available money correspond to a constraint]

ii Guess the definitions of marginal productivity of labour, marginal product of capital [or refer to the end of Example 3 in 7.3 on page 372].

iii Verify that $\frac{\text{marginal productivity of labour}}{\text{marginal productivity of capital}} = \frac{\text{unit price of labour}}{\text{unit price of capital}}$.

4 [7.4.12, Distance from a point to a Parabola] Find the point on the parabola $y = x^2$ that has minimal distance from the point $(16, \frac{1}{2})$ [Hint: The distance d from (x,y) to $(16, \frac{1}{2})$ is given by $d = \sqrt{(x-16)^2 + (y-\frac{1}{2})^2}$. To minimize d it suffices to minimize d^2]