MATH 55 - Worksheet 1 (Thursday)

1 Show that the following statements about the real number x are all equivalent to one another:

a x is rational.

b $3x + 2$ is rational.

c $3x - 1$ is rational.

2 The Collatz Conjecture

Choose any positive integer n. Let T be the transformation that outputs $\frac{n}{2}$ if n is even and $3n+1$ if n is odd. Then, the conjecture states that repeatedly applying T will eventually give us 1.

Ex Consider $n = 12$: Then, we have $T(12) = 6$, $T(6) = 3$, $T(3) = 10$, $T(10) = 5$, $T(5) = 16$, $T(16) = 8$, $T(8) = 4$, $T(4) = 2$, $T(2) = 1$.

Verify the conjecture for the following integers

a 5

b 6

c 7

d 11

For some history and interesting facts refer to corresponding Wikipedia article.

2 Tiling Checkerboards

a Read and understand Example 18, Example 19 and Example 20 in 1.8.

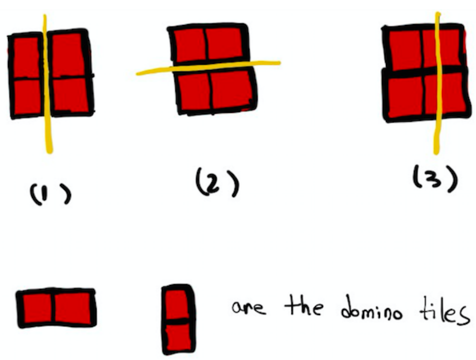
b Prove that you can use dominoes to tile the 8×8 checkerboard with two adjacent corners removed.

c Prove that you can use dominoes to tile any checkerboard with an even number of squares i.e. an $m \times n$ board with mn (the number of squares) being even.

d Prove that you can use dominoes to tile the standard checkerboard with all four corners removed. [Hint: refer to **b**]

The following two problems are harder, only attempt them if you want a challenge and are comfortable with the arguments used in parts **a** - **d**.

e Tile a checkerboard by dominoes. We say that the board is **separated**, if there is a line that cuts the checkerboard into two pieces and doesn't cut any domino. For simplicity, we will restrict to only vertical or horizontal lines. In the following diagrams, (1) and (2) is a separated board while (3) is not separated.



Notice that diagrams (1) and (2) prove that any tiling of 2×2 board has a separation (2×2 checkerboards have exactly 2 tilings). Prove, by using proof by cases/exhaustion that every tiling of a 4×4 board is **separated**.

f [**Challenge**] Prove that every tiling of a 6×6 board is separated. There's a solution to this that doesn't use cases/exhaustion.