MATH 16B - WORKSHEET 1

1 Let $f(x,y) = x^3 + x^2 + 4xy$

i Evaluate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}$

ii Evaluate the above partial derivatives at $(1,-2)$

iii Evaluate and compare $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

iv Find all points such that $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$.

v Compute the function $D(x,y) = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$. As you will see in the next lecture (?), $D(x,y)$ is related to the second derivative test for two variables functions.

2 Let $f(x, y, z) = \dfrac{x}{z - e^y}$.

i Evaluate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

** If you want more practice, evaluate $\frac{\partial^3 f}{\partial z \partial y \partial x}$

3 Consider the production function $f(x, y) = 60x^{\frac{3}{4}}y^{\frac{1}{4}}$ (x is the units of labour). Compute $\dfrac{\partial^2 f}{\partial x^2}$ and think about why it's always negative.