

Math 55 - Practice Midterm 1

Instructor - Ritvik Ramkumar

July 7, 2017, 10:20PM - 11:20PM, Etcheverry Hall 3111

Name: _____

SID: _____

Instructions:

- Write all answers in the provided space. Please write carefully and clearly, in complete English sentences.
- You are not allowed to use any notes, books, electronic devices, or your own scratch paper.
- Questions 2 – 5 require you to justify your answers

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Grade Breakdown

Question	Points	Maximum
1		15
2		10
3		5
4		5
5		18
Total		53

- 1 State whether each of the following statements are True or False. There is no need to provide an explanation, and no credit will be given for explanations!
- a (3 points) Every subset of an uncountable set is countable.
- b (3 points) $(p \vee q) \wedge (\neg p \vee r)$ and $q \vee r$ are logically equivalent
- c (3 points) Let A and B be finite sets with $|A| = |B|$. If $f : A \rightarrow B$ is injective (one-to-one), then it's surjective (onto).
- d (3 points) $n^3 \equiv n \pmod{4}$ for all $n \in \mathbf{Z}$.
- e (3 points) $\forall a, b \in \mathbf{Z}^+ \quad \exists c \in \mathbf{Z} \quad (a^2 + c^2 = b^2)$.

2a (8 points) Use the Euclidean algorithm to solve the congruence $7x \equiv 1 \pmod{11}$

2b (2 points) Solve the congruence $7x \equiv -2 \pmod{11}$.

3 (5 points) Find the largest negative solution to the system of equations: $x \equiv 8 \pmod{12}$ and $x \equiv 6^{85} \pmod{13}$.

4 (5 points) Prove that the function $f : \mathbf{Z} \times \mathbf{Z} \longrightarrow \mathbf{Z}$ given by $f(m, n) = 2m - 7n$ is surjective (onto).

5a (2 points) Define what it means for a set S to be countable.

5b (7 points) Let C and D be countable sets. Prove that $C \cup D$ is countable [Hint: Split it into three cases, depending on whether C or D is finite]

c You may use the result in **a** to justify your answers in the following parts. Find with proof examples of uncountable sets A, B such that

i (3 points) $A - B$ is finite

ii (3 points) $A - B$ is countably infinite

iii (3 points) $A - B$ is uncountable