Math 55 - Summer 2017, Midterm 1

Instructor - Ritvik Ramkumar

July 7, 2017, 10:20AM - 11:30AM, Etcheverry Hall 3111

Name: ____

SID: ____

Instructions:

- Write all answers in the provided space. Please write carefully and clearly, in complete English sentences.
- You are not allowed to use any notes, books, electronic devices, or your own scratch paper.
- **Questions** $2 - 6$ **require you to justify your answers**

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Grade Breakdown

Question	Points	Maximum
1		15
2		11
3		5
4		5
5		10
6		4
Total		**50**

1 State whether each of the following statements are True or False. There is no need to provide an explanation, and no credit will be given for explanations!

a (**3 points**) If $|A| = |B|$ and $|C| = |D|$, then $|A \times B| = |C \times D|$.

b (**3 points**) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

c (**3 points**) $\mathbf{R} - \mathbf{Q}$ is countable.

d (**3 points**) The function $f : \mathbf{Z} \times \mathbf{Z} \longrightarrow \mathbf{Z}$ given by $f(m, n) = 9n - 12m$ is onto.

e (**3 points**) If $\forall x \exists y P(x, y)$ is True, then $\exists x \forall y P(x, y)$ is True.

2a (**8 points**) Use the Euclidean algorithm to solve the congruence $9x \equiv 1 \mod 16$

2b (**3 points**) Solve the congruence $9x \equiv 3 \mod 16$.

3 The goal of this exercise is to compute $11^{10^5} \bmod 13$.

a (**3 points**) Prove that if r is an integer such that $10^5 \equiv r \bmod 12$ then $11^{10^5} \equiv 11^r \bmod 13$

b (**2 points**) Find a suitable r and use it to compute $11^{10^5} \bmod 13$.

4 (**5 points**) Determine whether $\forall x(P(x) \longrightarrow Q(x))$ and $\forall x P(x) \longrightarrow \forall x Q(x)$ are logically equivalent.

5

a (**5 points**) Let A, B be finite sets. Prove that a function $f : A \longrightarrow B$ is one-to-one iff $|f(S)| = |S|$ for all subsets S of A.

b (**2 points**) Find the number of distinct functions from $\{1, 2\} \longrightarrow \{1, 2\}$.

c (**3 points**) Prove that the set of functions from $\mathbf{Z}^+ \longrightarrow \mathbf{Z}^+$ is uncountable.

6 Theorem: For any $n \in \mathbf{Z}$, $49n + 5$ is never a product of **two** consecutive integers; equivalently, $49n + 5 = m(m+1)$ has no solutions for any $n, m \in \mathbf{Z}$

a (**2 points**) For which $m \in \mathbf{Z}$, is $m(m+1) \equiv 5 \bmod 7$? Give your answer modulo 7.

b (**2 points**) Using part a, or otherwise, prove the Theorem.

Bonus (**3 points**) Show that $49n + 5$ is never a product of two or more consecutive integers.