MATH 113 - HOMEWORK 1

Due in class on Thursday June 28, 2018

- Some of these problems have hints; I recommended you try to solve the problem by yourself and only read the hint if you've been stuck for some time.
- All problems of the form Section x: y are from Fraleigh's book.
- One usually denotes $\mathbf{R} - \{0\}$, $\mathbf{Q} - \{0\}$, $\mathbf{C} - \{0\}$ by write $\mathbf{R}^\star$, $\mathbf{Q}^\star$, $\mathbf{C}^\star$ etc.
- Given a real number n we can define $\mathbf{R}_{>n} = \{x \in \mathbf{R} : x > n\}$. Similarly we have $\mathbf{R}_{\geq n}$, $\mathbf{R}_{<n}$, $\mathbf{Q}_{\geq n}$ (here n is a rational number) and so on..

1. Section 4: **31, 32, 34**

2. Prove that if $(ab)^2 = a^2b^2$ for all $a, b \in G$, then G is abelian. [We suppress the $\star$ operation and just concatenate letters]

3. Let G be a group and $g \in G$ an element. Prove that the function $i_g : G \longrightarrow G$ given by $x \mapsto gxg^{-1}$ is an automorphism of G. This is called the *conjugation action*.

4. Section 5: **9, 10, 23, 41, 47**

5. Which of the following sets of functions $f : \mathbf{R} \longrightarrow \mathbf{R}$ forms a group under composition? Prove your answers. You may assume that the set of all bijections from $\mathbf{R}$ to $\mathbf{R}$ is a group.

(a) The set of all bijections from $\mathbf{R} \longrightarrow \mathbf{R}$ satisfying $f(1) = 1$.

(b) The set of all bijections from $\mathbf{R} \longrightarrow \mathbf{R}$ satisfying $f(1) = 2$.

(c) The set of all bijections from $\mathbf{R} \longrightarrow \mathbf{R}$ with the property that if $a > b$, then $f(a) > f(b)$.

(d) The set of all bijections from $\mathbf{R} \longrightarrow \mathbf{R}$ with the property that if $x > 0$, then $f(x) > 0$.

6. In Section 5, 47 you showed that $\{x \in G : x^2 = e\}$ forms a subgroup if G is abelian. Find a counterexample to this statement when G is not abelian.

7. Which of the following are homomorphisms of groups? Prove your answers.

(a) $f : (\mathrm{GL}_2(\mathbf{R}), \times) \longrightarrow (\mathbf{R}^\star, \times)$ defined by $A \mapsto \det(A^{-1})$.

(b) $f : (\mathbf{R}_{>0}, \times) \longrightarrow (\mathbf{R}, +)$ defined by $x \mapsto \ln x$

(c) $f : (\mathbf{Z}, +) \longrightarrow (\mathbf{Z}, +)$ defined by $n \mapsto n^2$.

Other problems, **NOT** to be handed in:

- Read Hutching's "Introduction to Mathematical Proofs"
- Section 4: 19 (a funny looking group), 20 (an outline of how to construct all groups on 4 elements)
- Section 5: 1 - 7 (simple examples of subgroups/non subgroups)