Math 113 - Summer 2018, Midterm

Instructor - Ritvik Ramkumar

July 18, 2018, 4:10PM - 5:30PM, Cory 289

Name: ______________________________

INSTRUCTIONS:

- Write all answers in the provided space. Please write carefully and clearly, in complete sentences.
- You are only allowed to use one sheet of paper with handwritten notes
- **Justify** all your answers. In particular, if you're using a theorem from class, clearly state it's name or contents.
- You're only allowed to use results I've stated in class or on a Homework.

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Grade Breakdown

Question	Points	Maximum
1		10
2		15
3		10
4		15
5		10
Total		**60**

1

a [5 points] Let G be an abelian group of order 200. List, up to isomorphism, all possible groups G. Which of the groups listed are cyclic?

b [5 points] Find two non-isomorphic abelian groups of order 200 that contain an element of order 100.

2 An element x of a group G is said to be a *square* if $\exists y \in G$ such that $y^2 = x$.

a [2 points] Find all the squares in $G = \mathbf{Z}/4\mathbf{Z}$.

b [3 points] Find all the squares in $G = \mathbf{Z}/5\mathbf{Z}$.

b [5 points] Let G be a finite, even order cyclic group, find all elements of G that are squares.

c [5 points] Let G be a finite, odd order cyclic group. Prove that every element of G is a square.

3

a [7 points] Let G be a group of order 62. Prove that every proper subgroup of G is abelian.

b [3 points] Is every group of order 62 abelian? If yes prove it, if not give a counterexample.

4 Let $X = \{(12)(34), (13)(24), (14)(23)\} \subseteq S_4$ be a set. Let S_4 act on X by conjugation i.e. $g(x) = gxg^{-1}$ (you may assume that this is an action).

a [5 points] Let ϕ_g be the permutation of X associated to g. Compute $\phi_{(12)}$, $\phi_{(13)}$ and $\phi_{(23)}$.

b [5 points] Show that there's a surjective group homomorphism $\Phi : S_4 \longrightarrow S_3$.

c [5 points] Consider the subgroup $N = \{e, (12)(34), (13)(24), (14)(23)\}$ of S_4. Prove that N is normal and show $S_4/N \cong S_3$.

5 *Isomorphic subgroups can have non-isomorphic quotients*

a [5 points] Let $G = (\mathbf{Z}/2\mathbf{Z}) \times (\mathbf{Z}/4\mathbf{Z})$. Find all subgroups of G of order 2.

b [5 points] Find two subgroups $H, K \subseteq G$ such that $H \cong K$ but $G/H \not\cong G/K$.