Math 113 - Abstract Algebra - Summer 2018

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Office hours : Wednesday, Thursday 3 - 4 pm, 1062 Evans Hall

Homework : Due every Thursday

Exams : Midterm on 7/17, Final on 8/9.

Grades : ~~20% [illegible] Assign~~ Midterm 40%, Final 40%, Hwk 20%.
(see syllabus for more info).

Lecture structure : I will lecture for about 1 hr 20 mins and then I will put some simple problems on the board to help internalize the material. You may also come up and ask me questions.

6/18 In this course we will study the "algebraic" aspects of mathematical structures you've seen before.

<u>Ex</u> ① $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, \ldots\}$ the integers

We know $\mathbb{Z}$ as a <u>set</u>, but it has more structure!
We can talk about addition of two integers (elements of $\mathbb{Z}$) and obtain an integer. This corresponds to a function

$+ : \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z}$ by $(n, m) \longmapsto n + m$.

There's multiplication $\times : \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z}$ by $(n, m) \longmapsto nm$.

② $\mathbb{Q} = \{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \}$ the rationals.

Along with addition and multiplication, there's an multiplicative "inversion" i.e. $\mathbb{Q} \longrightarrow \mathbb{Q}$ by $p \longmapsto p^{-1} = \frac{1}{p}$.

3 $\mathbb{R}$ = the real numbers. It has addition, multiplication and inversion.

One of the ~~[illegible]~~ goals of this course will be to formalize these operations.

There's a vague hierarchy, depending on the "amount" of operations one puts on a set:

$(\mathbb{R}, +)$ and scalar multiplication —— A vector space $/\mathbb{R}$ (one dimensional)

$(\_\_, +)$, anything with an operation like addition —— Group

$(\_\_, +, \times)$, anything with addition and multiplication —— Ring

$(\_\_, +, \times)$, addition, multiplication and inversion —— Field

The other goal is to apply the theory on some interesting examples.

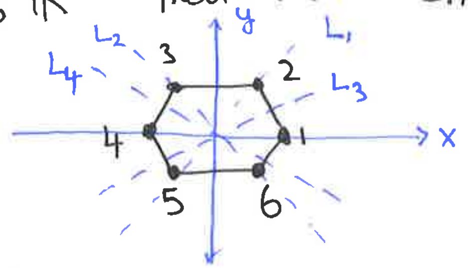
Ex You might have heard about the group of symmetries of a triangle Δ, square $\square$ etc. We will define these rigourously.

Ex You know of the quadratic "formula", and maybe you've seen the cubic, quartic formula, but you will not find a ~~quintic~~ quintic formula. We will ~~describe~~ formulate this famous result of Galois, and outline the steps of the proof. Unfortunately, we will not have developed enough theory to prove the result.

(2.5)

Ex Dihedral groups: Let C_n, for $n \geq 3$ denote a regular n-gon (embedded in $\mathbb{R}^2$). A dihedral symmetry is a rigid motion of $\mathbb{R}^2$ that moves C_n back to itself.
(sends vertices → vertices
edges → edges)

$n = 6$
C_6



- Rotations around the center $(0,0)$ (by multiples of $60°$)
- Flips (reflections) along the axes on L_1, L_2, L_3, L_4 (opposite vertices or center of sides)
- Identity symmetry.

The "group" of symmetries is denoted by D_n (think of it as a set for now)

Clearly D_n includes, n rotations by $2\pi/n$ radians and n reflections

So $|D_n| \geq 2n$.

Prop $|D_n| = 2n$

PF We only need to prove $|D_n| \leq 2n$. Let σ be a symmetry of C_n. Then σ must send 1 to another vertex $i \in \{1, 2, \ldots, n\}$. Since σ preserves edges, it must send 2 to $i+1$ or $i-1$. Thus there are only $n \cdot 2$ possibilities for $(\sigma(1), \sigma(2))$. Since $\sigma(1), \sigma(2)$ determine σ, we have $|D_{2n}| \leq 2n$ ∎

convince yourself of this (maybe a picture)

The group operation in action: