MATH 55 - HOMEWORK 4

Due in class on Monday July 17, 2017

6.1 14, 16, 22, 26, 40

6.2 3, 9, 12, 14

6.3 10, 12, 26, 28, 38

6.4 8, 10, 14, 21, 24

Challenge [Not to be handed in]

1 In class, we computed the number of functions and the number of injective functions from $\{1, \dots, n\}$ to $\{1, \dots k\}$. The goal of this exercise is to find the number of **surjective** functions.

Let $F(n, k)$ denote the number of surjective functions with domain $\{1, \dots, n\}$ and codomain $\{1, \dots, k\}$.

a Give a combinatorial proof of the following identity:

$$\sum_{k=0}^{m} \binom{m}{k} F(n,k) 2^{m-k} = \sum_{j=0}^{m} \binom{m}{j} j^n \qquad \text{for all } m, n \in \mathbf{N}.$$

Note A combinatorial proof of an identity is to find a **natural** bijection between two sets one having the cardinality equal to the LHS and the other having cardinality equal to the RHS.

b Adapt the argument from part a to prove, $F(n, m) = \sum_{j=0}^{m} (-1)^{m-j} \binom{m}{j} j^n$ [Hint: If you did part a in a non-roundabout way, the 2^{m-k} factor counts the number of subsets of some $(m-k)$-element set. Consider what happens if instead of treating all of these subsets the same, we "count" the odd-subsets with factor of -1]