

MATH 55 - HOMEWORK 2

Due in class on Monday July 3, 2017

2.2 3, 19, 32

2.3 6, 8, 12, 20, 44

2.4 26, 32, 34

2.5 4, 20

4.1 9, 16, 20, 35

4.2 4, 11, 32

4.3 24, 32, 40, 49, 52

4.4 6, 8, 12, 19, 21, 29, 30, 38

Challenges [Not to be handed in]

- 1 [2.5.27] Show that a union of a countable number of countable sets is countable. In set notation, this is asking to show that $\bigcup_{i \in I} A_i$ is countable where the index set I is countable and each A_i is countable. [Hint: Modify the proof that $A \cup B$ is countable whenever A, B are countable. It isn't a direct modification; you can't consecutively list the first element of each A_i]
- 2 Prove that a prime number p is a sum of two squares i.e. $p = a^2 + b^2$ with a, b integers, if and only if $p \equiv 1 \pmod{4}$. Follow these steps:
 - a Verify that 1, 5, 13, 17, 29 are all sums of squares.
 - b If $p \equiv 1 \pmod{4}$, use the results of Monday's lecture to conclude that $p = a^2 + b^2$.
 - c By studying the quadratic residues modulo 4, conclude that if $p = a^2 + b^2$, then $p \equiv 1 \pmod{4}$.