MATH 55 - HOMEWORK 1

Due in class on Monday June 26, 2017

1.1 12, 17, 48 [Hint: A proposition is either false or true, but not both]

1.3 25, 29 [Do not use the De-Morgan rules for these two exercises; the goal is to prove them]

1.4 14, 43

1.5 10, 20,

1.6 15

1.7 13,14 18, 24

1.8 3, 29, 34

2.1 7, 8, 23

Challenge **[Not to be handed in]** Exercise 25 in 1.8. Try to do it by yourself first; if you're stuck, follow these steps:

1 Run the process for $n = 3$ i.e. write down $1, \ldots, 6$ on the blackboard, and run the process until you have one integer left; verify that you are left with an odd integer. Repeat this once more (with different choices).

2 For any process, let $S(m)$ be the sum of the numbers on the blackboard, after step m. Compute $S(m)$ for all m in both the processes.

3 Is there something common between all the $S(m)$? Generalize this to any process.

Now try to prove your generalization as follows:

4 Compute the sum on the blackboard before the process begun i.e. compute $1 + 2 + 3 + \cdots + 2n$.

5 Notice that $S(m+1) - S(m) = |j - k| - j - k$, where j and k are chosen on the $m+1$-th step. What can you say about this, independent of j and k?

6 Combine **4** and **5** to deduce that the leftover integer is odd.