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APPLICATIONS OF STRONG MORITA EQUIVALENCE TO TRANSFORMATION GROUP C*-ALGEBRAS

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We will describe here ten related situations in which the concept of strong Morita equivalence illuminates the relationship between various transformation group C*-algebras. One feature which we will notice is that situations which may be very difficult to understand up to isomorphism may nevertheless become tractable if one is content to understand them only up to strong Morita equivalence.

There is some evidence, especially in the work of P. Green [4,5], that a considerable part of the material discussed below carries over to crossed products involving C*-algebras which need not be commutative, but the exact extent to which this is true is not clear at present. Important parts of the material discussed below are themselves due to P. Green, as will soon be apparent.

For the definition and general properties of strong Morita equivalence we refer the reader to [16] found elsewhere in these proceedings, or to [10,11,12,13]. If G is a locally compact group which acts as a transformation group on a locally compact space M, then G acts as a group of automorphisms of the commutative C*-algebra, $C_\infty(M)$, of continuous complex-valued functions which vanish at infinity. One can thus form the corresponding crossed product C*-algebra, as discussed earlier in these proceedings. In the present context this crossed product algebra is called the transformation group C*-algebra for the action of G on M, and is denoted by $C^*(G, M)$. The representations of $C^*(G, M)$ correspond to the

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covariant representations of G and $C_\infty(M)$, as discussed elsewhere in these proceedings, and $C^*(G, M)$ is the completion, with respect to the norm obtained from these representations, of the algebra $C_c(G, M)$ consisting of the continuous functions of compact support on $G\times M$ in which the product and involution are defined by

$$(\Phi*\Psi)(x, m) = \int_G \Phi(y, m)\Psi(y^{-1}x, y^{-1}m)dy,$$

$$\Phi^*(x, m) = (\Phi(x^{-1}, x^{-1}m))^{-}\Delta(x^{-1}),$$

for $\Phi, \Psi \in C_c(G, M)$, $x \in G$, $m \in M$. Most computations in this subject are first carried out for $C_c(G, M)$, and then extended to $C^*(G, M)$ by density. Below we will only indicate what happens for $C_c(G, M)$.

SITUATION 1. Let $M = G$ and let G act on M by left translation. Then it is well-known that $C^*(G, G)$ is isomorphic to the algebra of compact operators on $L^2(G)$. This is essentially the theorem on the uniqueness of the Heisenberg commutation relations [9]. Thus we do not need the concept of Morita equivalence to understand this situation (though it is related to the next situation by the fact that the algebra of compact operators is strongly Morita equivalent to the algebra of complex numbers). But this situation is a good place to start, as it helps to suggest what might happen in the next situation.

SITUATION 2. We generalize the above situation by allowing now many closed orbits, but we still require that the action of G be free, that is, we require that if $xm = m$ for some $m \in M$ and $x \in G$, then $x = e_G$. We, in fact, need to assume a bit more than that the orbits be closed, namely, we need to assume that compact sets are wandering, in the sense that for any compact subset K, of M, $\{x \in G: (xK)\cap K \neq \phi\}$ should be precompact in G. Since the orbits will look like G, we expect that $C^*(G, M) = A$ will look like a field of algebras of compact operators glued together in some way (a homogeneous C^*-algebra). But this glueing process can be complicated, so it seems hopeless to find any general statement describing precisely the isomorphism class of $C^*(G, M)$. However one can find a nice statement concerning the Morita equivalence class, namely that $C^*(G, M)$ is always

Morita equivalent to $C_\infty(M/G)$, where M/G is the orbit space with quotient topology, which will be locally compact Hausdorff by the wandering condition. This result, and more, is basically contained in [3]. For the equivalence bimodule one takes $X = C_c(M)$, with elements of $B = C_\infty(M/G)$ viewed as functions on M acting by pointwise multiplication on X. The B-valued inner product is given by

$$\langle f, g\rangle_B(m) = \int_G \bar{f}(x^{-1}m)g(x^{-1}m)dx.$$

The left translation action of G on X by

$$(\alpha_x(f))(y) = \Delta(x)^{\frac{1}{2}} f(x^{-1}y)$$

is "unitary" for the B-valued inner product, and together with the action of $C_\infty(M)$ on X by pointwise multiplication, gives a "covariant representation" of $(G, C_\infty(M))$ whose integrated form is an action of A on X. Now one property of an A-B-equivalence bimodule is that

$$\langle f, g\rangle_A h = f\langle g, h\rangle_B \quad \text{for } f, g, h \in X.$$

This property determines $\langle\ ,\ \rangle_A$ in terms of $\langle\ ,\ \rangle_B$. Using this fact in the present situation, we find by a simple calculation that

$$\langle f, g\rangle_A(x, m) = \Delta(x)^{-\frac{1}{2}} f(m)\bar{g}(x^{-1}m).$$

We give a specific example. Let G be the two-element group, acting on the sphere, M, by the antipodal map, so that M/G is real projective space. Then $C^*(G, M)$ corresponds to a field of 2×2 matrix algebras, M_2, over M/G. But simple computations show that $C^*(G, M)$ is not isomorphic to $C(M/G)\otimes M_2$, so that the description of the isomorphism class of $C^*(G, M)$ is a bit complicated.

SITUATION 3. In topological dynamics, if one has a homeomorphism, h, of a space P, and so an action of the integers, Z, on P, then a standard construction is to form the corresponding "flow under a constant function". For this one lets $M = (P\times[0,1])/\sim$, where $\sim$ is the equivalence relation on top and bottom given by $(p,1) \sim (h(p), 0)$. One then lets R act on M by letting a point move up the vertical fibre until it reaches height 1, so that the point is of form $(p, 1)$, at which time one skips to $(h(p), 0)$ and continues rising along that fibre.

In this situation $C^*(R, M)$ is strongly Morita equivalent to $C^*(Z, P)$. This fact, as well as the above construction, is a special case of:

SITUATION 4. Let H be a closed subgroup of G, and let H act on the space P. Let $M = G\times_H P = (G\times P)/\sim$, where $\sim$ is the equivalence relation $(xs, p) \sim (x, sp)$ for $x \in G$, $s \in H$, $p \in P$. Let the action of G on M be that which comes from the action of G on G by left translation. Then $C^*(G, M)$ is strongly Morita equivalent to $C^*(H, P)$. This will be seen to be a special case of situation 7 below.

We give a specific example. Let P be the unit circle, and let Z act on P by powers of some fixed irrational rotation (a free action, but with non-closed orbits, so not wandering). Then $C^*(Z, P)$ is known to be a simple antiliminal algebra with identity. If we view Z as a subgroup of R, then it is easily seen that $M = R\times_Z P$ is the torus, and the action of R is just a flow at an irrational angle. Then $C^*(R, M)$, which is a simple antiliminal algebra without unit, is Morita equivalent to $C^*(Z, P)$. This was pointed out to me by Phil Green.

SITUATION 5. We now turn to situations in which the action is explicitly not free. The simplest such occurs when $M = G/H$ for some closed subgroup H, with G acting by left translation. The covariant representations are essentially the "transitive systems of imprimitivity" occuring in induced representations, and Mackey's imprimitivity theorem essentially is the fact that $C^*(G, G/H)$ is strongly Morita equivalent to $C^*(H)$. All this is discussed in [10]. The equivalence bimodule is $C_c(G)$. If G and H are unimodular then $C_c(H)$ acts on the right by convolution, and $C_c(G\ G/H)$ acts on the left as the "integrated form" of the evident "covariant representation" of G and $C_\infty(G/H)$ on $C_c(G)$. The inner products are defined by

$$\langle f, g\rangle_A(y, \dot{x}) = \int_H f(xt)g^*(t^{-1}x^{-1}y)\,dt$$

$$\langle f, g\rangle_B(t) = (f^* * g)(t).$$

In the non-unimodular case all this must be decorated with modular functions.

SITUATION 6. Let G act on a space P, let H be a closed subgroup of G, so that also H acts on P, and let G act on $M = (G/H)\times P$ by the diagonal action. Then $C^*(G, (G/H)\times P)$ is strongly Morita equivalent to $C^*(H, P)$. (For $G = R$, $H = Z$, P = circle with R giving a flow at an irrational rate, one obtains the specific example in 4 above.) Clearly 5 above is a special case of this situation, which in turn is a special case of:

SITUATION 7. Let G act on M, let H be a closed subgroup, and suppose that there is a G-equivariant map, π, of M onto G/H. Let $P = \pi^{-1}(\dot{e})$, so that H acts on P. Then $C^*(G, M)$ is strongly Morita equivalent to $C^*(H, P)$. This is just a special case of theorem 17 of [4], where the setting involves actions of groups on not necessarily commutative C^*-algebras. (Some corresponding isomorphism theorems are contained in [5].) It is easily seen that if in 4 above one defines π by $\pi((x, p)^{\cdot}) = \dot{x}$, then 4 is a special case of the present situation, which in turn will be seen to be a special case of situation 10 below.

SITUATION 8. As in situation 3 above, let h be a homeomorphism of P, so that Z acts on P. Let F be any strictly positive continuous function on P. Then one can construct the "flow under the function F", in which M is the quotient of $\{(p, r) \in P\times R : 0 \le r \le F(p)\}$ by the equivalence relation $(p, F(p)) \sim (h(p), 0)$ and R acts on M as in 3. Again $C^*(R, M)$ is strongly Morita equivalent to $C^*(Z, P)$. This is a special case of situation 10 below.

SITUATION 9. Let H and K be two closed subgroups of G, so that we can let K act on G/H by left translation, and H act on $K\backslash G$ by right translation. Then $C^*(K, G/H)$ and $C^*(H, K\backslash G)$ are strongly Morita equivalent. This is the content of [12]. We give a specific example. Let $G = R$, $H = Z$, $K = Z\alpha$ for some irrational number α. Then $C^*(Z\alpha, R/Z)$ is the C^*-algebra for Z acting on the circle by irrational angle $2\pi\alpha$, while $C^*(z, R/Z\alpha)$ can be seen to correspond to rotation by $2\pi/\alpha$. These two simple antiliminal C^*-algebras are thus strongly Morita equivalent. However, by using techniques of K-theory it has recently been shown [2,7,8,15] that these two algebras are not isomorphic, and that, in fact, if α and β are any two irrational numbers between 0 and 1/2 then the corresponding algebras are isomorphic only if $\alpha = \beta$, though they will be strongly Morita equivalent exactly if α and β are in the same orbit for the action of $GL(2, Z)$ on the irrational numbers by linear fractional transformations [15].

By using the strong Morita equivalence of $C^*(K, G/H)$ with $C^*(H, K\backslash G)$, one can also show that if α is a rational number, then $C^*(Z\alpha, R/Z)$ is strongly Morita equivalent to $C(T^2)$, the algebra of continuous functions on the torus. Nevertheless, it has recently been shown [5] that for different rational numbers between 0 and 1/2 these algebras are still non-isomorphic. This result and the Morita equivalence in the irrational case described above, as well as the construction given by Connes [1] of certain modules over irrational rotation algebras, can, in fact, be illuminated by applying the present general situation to the case in which $G = R \times Z_q$ for various $q \in Z$ and in which H and K are various suitable cyclic subgroups of G.

SITUATION 10. Let H and K be any two locally compact groups, and let them both act on a space P with the two actions commuting, and with both actions free and wandering as in situation 2 above. Then $C^*(K, P/H)$ and $C^*(H, P/K)$ are strongly Morita equivalent. This is an unpublished result of Phil Green, and with his kind permission we will include the proof here. But before doing so, let us indicate why all of the earlier situations are special cases of the present ones. To begin with, it is clear that this is the case for situations 2 and 9 above. But from the comments made earlier, it suffices to prove that this is also the case for situations 7 and 8.

For situation 7 this can be seen by letting P be the pull-back of π with the natural map of $G \to G/H$. That is, $P = \{(x, m) \in G\times M: \pi(m) = \dot{x}\}$, where the left action of $G = K$ on P is given by $y(x, m) = (yx, ym)$, while the action of H on P is given by $s(x, m) = (xs^{-1}, m)$. Then P/G is identified with $\pi^{-1}(\dot{e})$ by $(x, m) \to x^{-1}m$, while P/H is identified with

M by $(x, m) \to m$.

It was Paul Muhly who showed me that situation 8 could be viewed as a special case of situation 10 as follows. With P as in 8, let $Q = R \times P$ play the role of the P above. Let $H = Z$ and $K = R$. Define a cocycle, c, on $Z \times P$ by $c(0, p) = 0$, $c(n, p) = -\Sigma(F(h^k(p)) : 0 \le k \le n-1)$ if $n \ge 1$, and $c(n, p) = \Sigma(F(h^k(p)) : n \le k \le -1)$ if $n \le -1$. Let $H = Z$ act on Q by

$$n(r, p) = (r + c(n, p), h^n(p))$$

for $n \in Z$, $r \in R$, $p \in P$, and let R act on P by

$$t(r, p) = (r+t, p).$$

The action of H can be seen to be wandering because of the strict positivity of F.

Proof for situation 10. The proof can be considered to be a combination of the wandering techniques developed in [3] together with the techniques used in [12] to treat situation 9 above. As mentioned earlier, the wandering hypothesis is easily seen to imply that P/H and P/K are Hausdorff (locally compact). Let $A = C_c(K, P/H)$ and $B = C_c(H, P/K)$ or, at the end, their C*-completions. Let $X = C_c(P)$. We make X into an A-B-equivalence bimodule as follows. Let K and H act on the left and right of X by

$$(kf)(p) = \Delta_K(k)^{\frac{1}{2}} f(k^{-1}p)$$

$$(fs)(p) = \Delta_H(s)^{-\frac{1}{2}} f(sp)$$

for $f \in X$, $k \in K$, $s \in H$ and $p \in P$, where Δ_K and Δ_H are the modular functions of K and H respectively. Let $C_\infty(P/K)$ and $C_\infty(P/H)$ act on X by pointwise multiplication. If we let K act on $C_\infty(P/H)$ by ordinary translation, then it is easily verified that the representation on X of the pair $(K, C_\infty(P/H))$ is "covariant" and so gives a representation of A on X by

$$(\Phi f)(p) = \int_K \Phi(k, p) \Delta_K(k)^{\frac{1}{2}} f(k^{-1}p)\, dk$$

for $\Phi \in A$, $f \in X$ and $p \in P$. In the same way we obtain a right action of B on X by

$$(f\Omega)(p) = \int_H f(sx)\Omega(s, sx)\Delta_H(s)^{-\frac{1}{2}} ds$$

for $f \in X$, $\Omega \in B$ and $p \in P$. It is easily verified that these actions of A and B on X commute, so that X is an A-B-bimodule. We now define A- and B-valued inner products on X by

$$<f, g>_A(k, \dot{p}) = \Delta_K(k)^{-\frac{1}{2}}\int_H f(s^{-1}p)\bar{g}(k^{-1}s^{-1}p)ds$$

$$<f, g>_B(s, \ddot{p}) = \Delta_H(s)^{-\frac{1}{2}}\int_K \bar{f}(k^{-1}p)g(k^{-1}sp)dk$$

for $f, g \in X$, $s \in H$, $k \in K$, $p \in P$. Then all the properties of an A-B-equivalence bimodule are verified by routine calculations except the following:

a) the positivity of the above inner products
b) the density of the span of the range of the inner products
c) the continuity of the module structures with respect to the inner products.

To verify these properties we must construct approximate identities for A and B of a very special kind. Since the roles of A and B are almost symmetric, we will carry out the construction only for A. Then what we must show for A is that it has an approximate identity all of whose elements are sums of terms of form $<f, f>_A$ for various $f \in X$.

LEMMA. Let K be a locally compact group which acts on a locally compact space P such that the action is free and wandering. Then for each $p \in P$ and each neighbourhood N of the identity element of K there is a neighbourhood U of p such that

$$\{k \in K : kU \cap U \neq \phi\} \subseteq N.$$

Proof. Let N and p be given. If the lemma is not true for N and p, then for each neighbourhood U of p there are $k_U \in K$ and $p_U \in U$ such that $k_U \notin N$ but $k_U p_U \in U$. Fixing some compact neighbourhood, D, of p, we can restrict attention to the $U \subseteq D$. Then $k_U D \cap D \neq \phi$ for all such U, so that by the wandering hypothesis there must be a compact set C in K such that $k_U \in C$ for each U. By the compactness of C we can pass to a subnet, still denoted by k_U, which converges to an element, say k, of K. Since N is a neighbourhood of e_K and $k_U \notin N$ for each U, it follows that $k \neq e_K$. But p_U clearly converges to p, as does $k_U p_U$. It follows that $kp = p$, which contradicts the freeness of the action. Q.E.D.

Now it is easily verified that one has an approximate identity for A if one has a net, $\Phi_{N,D,\varepsilon}$ indexed by decreasing neighbourhoods, N, of e_K, increasing compact subsets D of P/H, and decreasing $\varepsilon > 0$, which satisfies:

1) $\Phi_{N,D,\varepsilon}(k, \dot{p}) = 0$ if $k \notin N$, and ≥ 0 otherwise,

2) $|\int_K \Delta(k)^{\frac{1}{2}} \Phi_{N,D,\varepsilon}(k, \dot{p})dk - 1| < \varepsilon$ for $\dot{p} \in D$.

In fact, such a net will be an approximate identity for the inductive limit topology on A, and so for both the C*-norm on A and the action of A on X.

We now construct such a net in a special way. Let N, D and ε be given. Let π denote the projection of P onto P/H, and choose a compact subset C of P such that $\pi(C) \supseteq D$. By the above lemma we can find a finite open covering, U_i, of C such that for each i

$$\{k \in K : kU_i \cap U_i \neq \phi\} \subseteq N.$$

Then for each i we can find $h_i \in C_c^+(U_i)$ such that Σh_i is strictly positive on C. Let

$$F(\dot{p}) = \Sigma \int_H h_i(s^{-1}p)ds,$$

so that F is strictly positive on D. Let

$$m = (\inf \text{ of } f \text{ on } D)/2 \quad \text{and} \quad G = \sup(F, m),$$

and let $f_i(p) = h_i(p)/G(\dot{p})$, so that $f_i \in C_c^+(P)$. Then we see that

$$\Sigma \int_H f_i(s^{-1}p)ds = 1 \quad \text{on } D, \text{ and is between 0 and 1 elsewhere.}$$

Furthermore, we see that

$$\{k : k(\text{support}(f_i)) \cap (\text{support}(f_i)) \neq \phi\} \subseteq N.$$

In what follows this will insure that condition 1 is met.

To motivate the next step, suppose that we had $\Phi = \Sigma\langle g_i, g_i\rangle_A$ for certain $g_i \in X$. In trying to verify condition 2 above we would calculate

$$\int_K \Delta(k)^{\frac{1}{2}} \Phi(k, \dot{p})dk = \Sigma \int_H (g_i(s^{-1}p)\int_K \bar{g}_i(k^{-1}s^{-1}p)dk)ds.$$

From this and the construction of the f_i we see that if we could find g_i such that

$$f_i(p) = g_i(p)\int_K g_i(k^{-1}p)dk,$$

with, in particular, $\text{support}(g_i) \subseteq \text{support}(f_i)$, then our goal would be attained. However, consideration of this equation shows that it can not always be solved for g_i. Nevertheless it can be solved approximately, and it is easily seen that this is sufficient (which explains the presence of the ε in the above).

LEMMA. Functions of the form $g(p)\int_K g(k^{-1}p)dk$ for $g \in C_c^+(P)$ are dense in $C_c^+(P)$ for the inductive limit topology.

Proof. Let $f \in C_c^+(P)$ and $\delta > 0$ be given. Define F on P/K by $F(\dot{p}) = \int_K f(k^{-1}p)dk$. Let $C = \{p \in P: f(p) \geq \delta\}$ and let $\dot{C}$ be the image of C in P/K. Note that $F(\dot{p}) > 0$ for $p \in C$. Let $m = \inf\{F(\dot{p}): p \in C\}$. Since $\dot{C}$ is compact, $m > 0$. Let $U = \{\dot{p}: F(\dot{p}) > m/2\}$, which is a neighbourhood of $\dot{C}$, and choose $Q \in C_c(P/K)$ such that $0 \leq Q \leq 1$, $Q(\dot{p}) = 1$ for $p \in C$ and $Q(\dot{p}) = 0$ for $\dot{p} \notin U$. Then $Q/((F^{\frac{1}{2}}) \in C_c(P/K)$. Let $g = fQ/(F^{\frac{1}{2}})$. Then $g \in C_c^+(P)$ and $\text{support}(g) \subseteq \text{support}(f)$. Furthermore, a simple calculation shows that

$$|f(p) - g(p)\int_K g(k^{-1}p)dk| \leq \delta \quad \text{for all } p. \qquad \text{Q.E.D.}$$

We now turn to the proofs of the three properties listed earlier as being still needed for the verification that X is an A-B-equivalence bimodule. Throughout we will denote an element of the approximate identity constructed above simply by $\Phi = \Sigma\langle g_i, g_i\rangle_A$.

a) Positivity. For any $f \in X$ it is easily seen that $\langle\Phi f, f\rangle_B$ converge to $\langle f, f\rangle_B$ in the inductive limit topology and so in the C*-norm. But

$$\langle\Phi f, f\rangle_B = \Sigma\langle\langle g_i, g_i\rangle_A f, f\rangle_B$$

$$= \Sigma\langle g_i\langle g_i, f\rangle_B, f\rangle_B = \Sigma\langle g_i, f\rangle_B^*\langle g_i, f\rangle_B,$$

which is clearly positive.

b) Density. If $\Psi \in A$ then $\Psi * \Phi$ converges to Ψ. But $\Psi * \Phi = \Sigma\langle \Psi g_i, g_i\rangle_A$

c) Continuity. The existence of the special approximate identity is used only in the form of the positivity proved above. We indicated earlier that the action of A on X comes from the corresponding "covariant" representation of K and $C_\infty(P/H)$

on X. A simple computation shows that the action of K on X is "unitary" for $<, >_B$. If $F \in C_\infty(P/H)$, then the fact that $\|f\|^2 - F^*F$ has a square root which is a bounded continuous function on P/H shows by a standard argument (using positivity) that the action of $C_\infty(P/H)$ is a norm-decreasing action by operators which are "bounded" with respect to $<, >_B$. Passing to the integrated form (and using states of B), we obtain the desired continuity.

By reversing the roles of A and B one obtains the remaining desired properties.

Recently D. Williams has obtained fairly general results concerning when transformation group C*-algebras are C.C.R (liminaire) [17] and when they are continuous trace algebras [18]. The techniques he uses are closely related to those described above, involving strong Morita equivalence and suitable wandering hypotheses, but he must also use hypotheses to the effect that the stability subgroups vary in a suitably continuous way. Thus his results do not fall within the framework described above.

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