1. For any $n \times n$ real matrix $T$ define an action $\alpha$ of $\mathbb{R}$ on the group $\mathbb{R}^n$ by $\alpha_t = \exp(tT)$ acting in the evident way. Let $G = \mathbb{R}^n \times_{\alpha} \mathbb{R}$. Then $G$ is a solvable Lie group. For the case of $T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ determine the equivalence classes of irreducible unitary representations of $G$, i.e. the irreducible representations of $C^*(G)$. Determine the topology on $\text{Prim}(C^*(G))$. Draw a picture. Discuss whether $C^*(G)$ is CCR or GCR, and why.

As a “warm-up exercise”, try first the simpler case for which $T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

These examples can give you a hint of the complexities of the representation theory of solvable Lie groups, and of the techniques used to deal with them.