Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}$ for some $s, t \in \mathbb{R}$, viewed as elements of the C*-algebra $M_2(\mathbb{C})$.

1. Determine for which $s, t$ we have $B \geq A$.

2. Determine for which $s, t$ we have $B \geq A^+$ (the positive part of $A$).

3. Find values of $s, t$ such that $B \geq A$, $B \geq 0$, and yet it is false that $B \geq A^+$. (So be careful about making false proofs.)

4) Find values of $s, t$ such that $B \geq A^+$ and yet it is false that $B^2 \geq (A^+)^2$. (So again be careful about making false proofs.)

5) Can you find values of $s, t$ such that $B \geq A^+$ and yet it is false that $B^{1/2} \geq (A^+)^{1/2}$?

6) For $2 \times 2$ matrices $T$ and $P$ such that $T \geq 0$ and $P$ is an orthogonal projection (i.e. $P^2 = P = P^*$), is it always true that $PTP \leq T$?