M. Rieffel Math 202A, Midterm Exam
October 24, 2019

INDICATE EXPLICITLY WHAT FACTS FROM THE COURSE YOU ARE USING.

SHOW YOUR WORK COMPLETELY AND NEATLY.

DON'T WRITE MORE THAN NECESSARY.

Total points = 30.

Name ....

SID# ....

Scores:

problem 1 ....

problem 2 ....

problem 3 ....

problem 4 ....

1. Let Λ be some index set (possibly infinite). For each $\lambda \in \Lambda$ let $(X_\lambda, \mathcal{T}_\lambda)$ be a topological space, and let $X = \Pi_\lambda X_\lambda$ be their product.

a) (2 points) Define what is meant by the product topology, $\mathcal{T}$, on X.

b) (4 points) Prove that if each $(X_\lambda, \mathcal{T}_\lambda)$ is Hausdorff then $(X, \mathcal{T})$ is Hausdorff.

2. a) (1 point) Define what is meant by a *homeomorphism* between two topological spaces.

b) (6 points) Let X be a compact topological space, let Y be a Hausdorff topological space, and let f be a continuous function from X to Y. Prove that if f is bijective (i.e. one-to-one and onto), then f is a homeomorphism.

3. Let $(X, \mathcal{T})$ be a topological space, and let E be an equivalence relation on X. Thus E is a subset of $X \times X$, and we can write $x \sim y$ if $(x, y) \in E$. Let $X/\sim$ be the set of equivalence classes for the equivalence relation E.

a) (1 points) Define what is meant by the quotient topology on $X/\sim$.

b) (7 points) prove that if the quotient topology on $X/\sim$ is Hausdorff then E is closed in $X \times X$ for the product topology.

4. (9 points) Let (X, d) be a compact metric space. Equip $C(X)$ with the supremum norm $\|\cdot\|_\infty$. For each $f \in C(X)$ let $L(f)$ be its Lipschitz constant with respect to d, defined by

$$L(f) = \sup\{|f(x) - f(y)|/d(x, y) : x, y \in X,\ x \neq y\}$$

(possibly $= +\infty$). Let

$$D = \{f \in C(X) : \|f\|_\infty \leq 1 \text{ and } L(f) \leq 1\}.$$

Prove that D is a compact subset of $C(X)$ for the metric from $\|\cdot\|_\infty$. (You may use major results proved in the course, but you should give a precise statement of such results.)