Math 55: Discrete Math

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Midterm #2 Review Sheet

1) A hand of three cards is drawn from an ordinary deck. If it is known that two of the cards are the ace of spades and the ace of clubs, what is the probability that all three cards are aces?

2) Stan wears either a bathing suit alone, or else a bathing suit, a T-shirt, and a hat. He has 6 bathing suits, 9 T-shirts, and 7 hats. In how many different ways can he get dressed?

3) In how many ways can I divide a class of 15 students into five groups of three students?

4) In Problem **3)**, given that Bob and Debbie are in the same group, what is the probability that Bob and Martha are in the same group? Given that Bob and Debbie are in the same group, what is the probability that Tim and Sue are in the same group?

5) Find the number of solutions to $x + y + z = 10$ in non-negative integers which satisfy $x \leq 5$ and $y \leq 3$.

6) A hand of three cards is drawn from an ordinary deck. It if is known that at least two of the cards are aces, what is the probability that all three cards are aces?

7) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Prove that any subset of S with six elements must contain two elements whose sum is 10.

8) Eight people check both a hat and a coat. The hats and coats are returned to the people at random. What is: a) the probability that every person gets their hat and coat back, b) the probability that no person gets either their hat or coat back, c) the probability that no one gets both their hat and coat back. Are getting one's hat back and getting one's coat back independent events?

9) You run into a person named Monte. He asks you to play a shell game; you agree. Monte places a bean at random under one of three shells. You pick the shell which you think hides the bean. Monte exposes one of the other two shells that does not hide the bean. What is the probability that the shell you picked hides the bean? Should you switch to the remaining shell?

10) If $\#(A \cup B) = 23,\ \#(C - A) = 12,\ \#(B \cap C) = 5,\ \#(A \cap B \cap C) = 1$, what is $\#(A \cup B \cup C)$?

11) License plates are 7-letter words, using only the 26 letters of the alphabet. How many license plates contain 'MOM'? 'DAD'? Both? Are these independent events?

12) A "triple-double" in poker is defined as a pair, plus three other cards in a row. We do not exclude the possibility of having three-of-a-kind. How many triple-double hands are there? Given that we have a triple-double, what is the probability that we have three-of-a-kind?

13) There are seven types of bagels at the store: rye, salt, wheat, peanut, corn, blue, and awful. The store has unlimited amounts of each kind of bagel. We purchase 10 bagels. Let A be the event that we get at least 2 rye bagels. Let B be the event that we get at most 3 salty bagels. Are A and B independent events? How about if we only buy one bagel?

14) There are eight colors, and we have three balls of each color. How many ways are there to choose four balls a) with repitition allowed, b) without repitition allowed? In either case, how many yellow balls can we expect to pick (yellow is one of the eight colors)?

15) How many ways are there to rearrange the letters in REARRANGE into two separate words? (such as: GREAN REAR)