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How many bit strings of length 18 have 13 0's and 5 1's?

A typical such string:

$$000100110000101000 = ***|**||****|*|***.$$

This is "stars and bars."

How many solutions are there to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 13$$

in which the x_i are non-negative integers?

How many ways are there to buy a dozen (= 13) bagels if there are six kinds of bagels available (and lots of each kind)?

How many solutions are there to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 19$$

in which the x_i are positive integers?

In how many ways can we divide a group of 10 students into 5 teams each with 2 members?

One answer:

$$\frac{\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}}{5!} = \frac{45 \cdot 28 \cdot 15 \cdot 6}{120} = 945.$$

Another way: $9 \cdot 7 \cdot 5 \cdot 3 = 945$.

How many ways to divide a group of 250 students into 125 teams of 2 members each?

$$1 \cdot 3 \cdot 5 \cdots 247 \cdot 249.$$

How many ways are there to divide a group of 250 students into 50 teams with 5 members each?

How many ways are there to distribute 5-card poker hands to a group of three players?

Answer: $\binom{52}{5}\binom{47}{5}\binom{42}{5}$.

Three poker hands will be extracted from a deck of 52 cards and then placed in three identical envelopes, which will be dropped into a hat. How many ways to do this?

Answer:

$$\frac{\binom{52}{5}\binom{47}{5}\binom{42}{5}}{3!}.$$

A book example: How many strings can be made by reordering the letters in

"COMMITTEE"?

There are 2 M's, 2 T's, 2 E's, and 1 each of C, O, I.

Answer:

$$\binom{9}{2}\binom{7}{2}\binom{5}{2} 3 \cdot 2 \cdot 1 = \frac{9!}{2^3} = 45360.$$