Ribet Office Hours — 885 Evans

- December 5: 2–3:30 PM
- December 9: 11–12:30 PM
- December 12: 11–12:30 PM
- December 16: 11–112:30 PM

Course Syllabus

- Foundations, sets, logic and functions — 8%
- Algorithms and the integers — 15%
- Mathematical reasoning — 10%
- Counting and probability — 37%
- Relations and graphs — 30% (roughly 13% relations and 17% graphs)

Downplayed

- Chebyshev's inequality
- Dijkstra's algorithm

High Points of Course

This is not supposed to be an exclusive list of everything that we've done. It's here to remind you of the structure of the course.

- Kinds of functions and properties of "onto," "1-1"
- Countability
- $O(f(x))$ and $o(f(x))$
- Euclidean division algorithm and concrete applications
- Congruences, Ch. Remainder Theorem, RSA encryption

- Methods of proof, especially induction
- Recursive things, e.g., Fibonacci #s
- Simple counting techniques (sum versus product rules)
- Inclusion-exclusion, especially with three or more sets
- Pigeons
- Permutations and combinations, and mixtures
- Stars and bars

- Elementary probability, e.g., conditional probability, expected values, Bernoulli trials
- Lenstra concepts, examples and homework questions...
- Derangements (thought I'd mention it)
- Relations, especially partial orderings and equivalence relations (study Hasse diagrams, topological sort. max/min elements)
- Simple graph properties and examples (e.g., C_n, K_n, $K_{m,n}$)
- Euler circuits and paths
- Euler's formula $2 = v - e + r$

Old Final Exam

How many onto functions are there from a set with 7 elements to one with 3 elements?

What is the coefficient of a^7b^5 in the expansion of $(2a - b)^{12}$?

Pokerhontas drops the King of ♣, the King of ♡ and the Ace of ♠ into a large bag. She shakes the bag vigorously and then removes two of the cards without looking at them. (a) If it is known that one of the two cards is a king, what is the probability that both cards are kings? (b) If it is known that one of the two cards is the King of ♡, what is the probability that both cards are kings?

Let G be the multigraph with adjacency matrix $\begin{pmatrix} 0 & 3 & 3 & 2 \\ 3 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$. How many vertices does G have? How many edges? Does G have an Euler circuit?

Each box of C^{++} Cereal comes packed with a small plastic toy, which is shaped either like a "C" or like a "+." A given box contains a "+" with probability 2/3 and a "C" with probability 1/3. Suppose that I buy n boxes of cereal ($n \geq 3$). What is the probability that I have at least one "C" and at least two "+"s?

How many ways can n books be placed on k distinguishable shelves (a) if the books are indistinguishable copies of the same title? (b) if no two books are the same and the positions of the books on the shelves matter?

Consider the first 250 Fibonacci numbers: $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2, \ldots$, $f_{249} \approx 5 \times 10^{51}$. (a) Show that there are at least 84 of them which have the same remainder when divided by 3. (b) How many of them are divisible by 2?

Find an integer d such that $(M^{11})^d \equiv M \bmod 55$ for all M prime to 55.

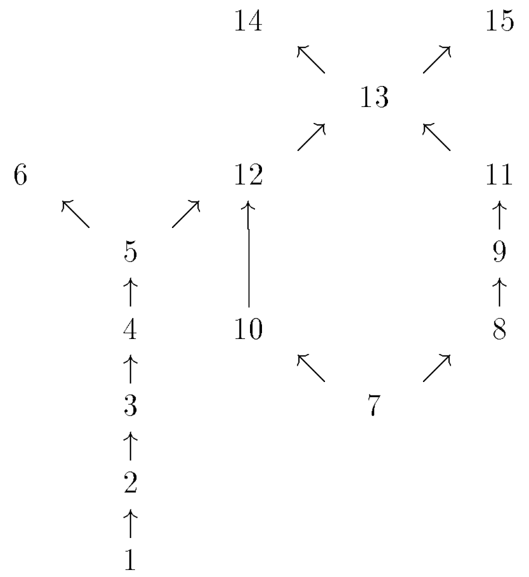
Show that $\frac{1}{\sqrt{n+1}} \geq 2\left(\sqrt{n+2} - \sqrt{n+1}\right)$ for all integers $n \geq 1$. (Multiply both sides by the positive quantity $\sqrt{n+2} + \sqrt{n+1}$.)

Use mathematical induction and the result of part (a) to show:

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \geq 2(\sqrt{n+1} - 1)$$

for $n \geq 1$.

The diagram below, copied from a famous number theory book, shows the logical relation among chapters — to understand chapter 12, for instance, you have to have read the first five chapters as well as chapters 7 and 10. The diagram defines a partial ordering on the set $S := \{1, 2, \ldots, 15\}$: $a \prec b$ if and only if a must be read before b. With respect to this partial ordering: (a) What is the least upper bound of 2 and 8? (b) What are the maximal and minimal elements of S? Finally, (c) describe a total ordering on S which is compatible with the given partial ordering.



Rejects

True or false: If $f(x) = \pi^x$ and $g(x) = 10^x$, then $f(x) = O(g(x))$.

How many bit strings of length 12 contain at most three 1's?

How many bit strings of length 10 contain at least 3 1s and at least 3 0s?

I reach into a drawer that contains my last four socks. These socks have been rolled into two "pairs": a true pair of black socks and a pair consisting of one black sock and one blue sock. I choose a pair at random and unroll it; then I examine one of the socks. It is black. What is the probability that the second sock in the chosen pair is black as well?

In how many ways can I buy 14 bagels if there are 5 kinds?

Josh, Oliver and Susan, in order, take turns tossing a fair coin: Josh goes first, and the game continues until someone wins by getting a "head." What is Susan's probability of winning?