Simplest situation: find the probability of an event E. It's the number of elements of E divided by the number of elements in the ambient set S, which often has to be inferred from the context.

"Find the probability of getting three of a kind in a poker hand." The set S is the set of all possible poker hands; it has $\binom{52}{5}$ elements. How many hands have three of a kind? There is a question of interpretation: we mean three of a kind but *not* four of a kind and not a full house. The number of such hands is $13 \cdot \binom{4}{3}\binom{48}{1}\binom{44}{1}$—I hope.

It's very easy to make errors in this subject. [*Addendum:* which I did! The correct number is half the one I gave, since there is no specific "fourth" or "fifth" card.]

More generally, take S to be finite, but allow the different elements x of S to have different probabilities $p(x)$. Then p is a function $S \to [0,1]$, and we want $\sum p(x) = 1$, where the sum is taken over all $x \in S$.

If E is a subset of S (an "event"), we set

$$p(E) = \sum_{x \in E} p(x).$$

Thus we may write $p(S) = 1$ with this notation.

Two easy formulas:

$$p(\overline{E}) = 1 - p(E) \quad \text{if } \overline{E} = S \setminus E.$$

$$p(E \cup F) = p(E) + p(F) - p(E \cap F).$$

The second follows from inclusion-exclusion. You can make up your own formula for the union of n events.

Find the probability of getting at least one "1" in a roll of two dice.

"A solution in non-negative integers to $x + y + z + w = 100$ is chosen at random. What is the probability that at least one of x, y, z, w is 4?"

Let S be the set of all solutions (x, y, z, w). It has $\binom{103}{3}$ elements (stars and bars). Let A be the set of solutions with $x = 4$; it has $\binom{98}{2}$ elements. Define B, C and D in the same way for y, z and w. We want to calculate the number of elements in $A \cup B \cup C \cup D$. By inclusion-exclusion, it's

$$\begin{aligned} &4 \cdot \#(A) - 6 \cdot \#(A \cap B) + 4 \cdot \#(A \cap B \cap C) \\ &= 4\binom{98}{2} - 6 \cdot 93 + 4. \end{aligned}$$

If E and F are events, the *probability of E given F* or $p(E|F)$ is $\dfrac{p(E \cap F)}{p(F)}$.

"Pokerhontas drops the King of ♣, the King of ♡ and the Ace of ♠ into a large bag. She shakes the bag vigorously and then removes two of the cards without looking at them. (a) If it is known that one of the two cards is a king, what is the probability that both cards are kings? (b) If it is known that one of the two cards is the King of ♡, what is the probability that both cards are kings?"

Answer to (a): Since she certainly removes at least one king, the "If it is known that…" gives no information! In symbols, $p(F) = 1$ and $E \cap F = E$. What is the probability of E, the event that she pulls out the two kings? This is clearly 1/3.

Answer to (b): There are precisely two sets of two cards that have the King of ♡ in them. Here F has 2 elements. One of these sets has the other king. Hence the answer is 1/2.

Independence: E and F are said to be independent if $p(E \cap F) = p(E)p(F)$. This is roughly the same as either of the equations $p(E|F) = p(E)$ and $p(F|E) = p(F)$.

Are the following two events independent: getting at least one "1" in the roll of two dice and getting a total of "7" in the same roll?