Today is November 13, 1997.
Administrative comments:

1. We are going to be doing the Course Evaluation at the end of this session. So please be prepared to express your opinion at 4:45.

2. From now on, there are going to be lots of pictures. These won't appear on the slide summaries which are posted to the Net.

Graphs

Graphs come in several flavors. They all have *vertices* and *edges*. The edges might be directed ($\longrightarrow$) or undirected (—). Some edges might be loops at a single vertex. There may or may not be multiple edges between pairs of vertices.

A *simple graph* has undirected edges, no loops, no multiple edges. Here are two examples:

- The cycle C_n on n vertices ($n \geq 3$).

- The complete graph K_n on n vertices $n \geq 1$.

Multigraphs are like simple graphs, except that multiple edges between two vertices are now permitted. You can make up a story about computer networks in which multigraphs would summarize the relevant information.

Pseudographs are undirected graphs in which we allow loops as well as multiple edges. You might have a line from a computer to itself, for some reason.

Directed graphs are graphs in which the edges are arrows instead of curves lines. Loops are allowed, but multiple edges in the same direction between two vertices are forbidden. An example would be the directed graph associated to a relation R on a finite set: there's an edge from a to b if and only if $(a, b) \in R$.

Directed multigraphs allow loops and multiple edges. Anything goes.

Simple graphs are called *bipartite* if you can separate the vertices into two groups such that all edges connect a vertex in one group to a vertex in the other. The complete bipartite graph $K_{m,n}$ has $m + n$ vertices, divided into a group of m vertices and a group of n vertices. You connect all vertices in the first group to all vertices in the second group.

The cycle C_n is bipartite if n is even.

The Handshaking Theorem

In an undirected graph, the *degree* of a vertex v is the number of edges coming out of v. A loop at v counts double. The *Theorem* is that we have the identity

$$2\#(\text{edges}) = \sum_{v \in V} \deg v,$$

where V is the set of vertices of the graph.

The proof is basically evident, since each edge contributes "2" to the sum of all the degrees.

Corollary 1 *An undirected graph has an even number of edges with odd degree.*

You can make similar statements for directed graphs. For each vertex v, define $\deg^{-}(v)$ to be the number of edges coming in to v and $= \deg +(v)$ to be the number of edges going out of v. We then have

$$\sum_v \deg^{-}(v) = \#(\text{edges}) = \sum_v \deg^{+}(v).$$

OK, time for course evaluation!